# MST121 – 2002 Solutions

**Qn.1** (a) 3.3, 3.6, 3.9
(b) Arithmetic Progression
$x_n = 2.7 + 0.3n \quad n=1,2...$

(c) $x_n$ becomes indefinitely large and positive.

**Qn.2** (a) gradient of AD is $-2/10 = -1/5$ which = the gradient of BC.
(b) DC is perp to BC, so its gradient = +5.
Also it goes through (11,3) so eqn. is $y=5x-52$

**Qn.3** (a) (5,3) (b) $\sqrt{6^2+2^2} = \sqrt{40}$
(c) $(x-5)^2+(y-3)^2=10$
(d) Solve $(x-5)^2+(0-3)^2=10$, that is
$x^2-10x-24=0 \Rightarrow (x-6)(x-4)=0$
=> x=6 or x=4 so points are (4,0) and (6,0)

**Qn.4** (a) $f$ (i) neither (ii) many-one
$g$ (i) decreasing (ii) one-one
(b) $f$ (i) $f$ is many-one and so does not have an inverse. For example $f(1) = f(-1) = 4$, so $f^{-1}(4)$ is not single-valued and therefore cannot define a function.
$g$ (i) $x = 2 - ½y$ which is decreasing and one-to-one, and so an inverse function exists which is (ii) $g^{-1}(x) = 2 - ½x$.

**Qn.5** (a) 121.64, or say 122 to nearest integer.
(b) Graph will approach E=900 from below.

**Qn.6** (a) $\sum_{i=1}^{40} 3 - 2\sum_{i=1}^{40} i = 120 - 40.41 = -1520$
(b) (i) $a_n = 3n + \frac{1}{4n}$ $3n$ increases indefinitely so there is no limit.
(ii) for large $n$, 2 becomes insignificant so limit is $\frac{12n}{3n}$, that is 4.

**Qn.7 (2002 candidates)**
(a) (4,0) (7.4, 5.2)
(b) (3.4, 5.2)
(c) magnitude = $\sqrt{11.373 + 26.648} = 6.2$
direction = $56.8^0$ (n.b >1 d.p. precision should be used in part (c) calculations, although in this case it does not affect the final answers).

**Qn.8** (a) $\begin{pmatrix} 2 & -3 \\ 4 & -7 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} -7 & 3 \\ -4 & 2 \end{pmatrix}$

$= \begin{pmatrix} 3½ & -1½ \\ 2 & -1 \end{pmatrix}$

(b) (i) $\begin{pmatrix} 2 & -3 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 26 \end{pmatrix}$

(ii) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3½ & -1½ \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 12 \\ 26 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

**Qn.9** (a) 18x + 5cos(5x) ;const multiple and sum rules

(b) $4\ln t + 2e^{-t/2}$ +c ;const multiple and sum rules

**Qn.10** (a) Vol = Area of base x ht.
$= 2x^2h = 72\ 000$ so $h=36\ 000/x^2$
(b) $\frac{dS}{dx} = 8x - \frac{216000}{x^2}$ which = 0 when
$8x^3 = 216000$ i.e. when $x = 60/2 = 30$

(c) Corresponding $h = 36000/30^2 = 40$

**Qn.11** (a) Use $v^2=2as$,
$a = ½(15^2/45) = ½(225/45) = 2.5\ ms^{-2}$
(b) Use $s = ½at^2$
$t^2 = 2s/a = 90/2½ = 36$ so time is 6 secs.
(or use v=u+at: 0=-15+2.5t to give t=6)

**Qn.12** (a) rising exponential curve starting at (0,15) and asymptotic to theta = 85.
(b) A=85, B=70
(c) 5 = 70.exp(-0.04t)
ln(1/14)=-0.04t
t=25.ln(14) = 65.98 (mins.)

**Qn.13** (a) 1/8 (b) 1/64
(c) $(63/64)^{12}$ =0.828
(d) 1 – 0.828 = 0.172
(e) mean number of rolls is 1/(1/64) = 64

**Qn.14** (a) The distribution is right skewed.
(b) Sketch a smoothed curve approximating the outline of the tips of the histogram bars.
(c) Use the area under this curve to the right of the line x=20, after adjusting the vertical scale is adjusted so total area under curve = 1.

**Qn.15** standard error = $2.4/\sqrt{40}$
1.96 times s.error = 0.744 so conf. interval
= (96.4 – 0.744, 96.4 + 0.744)
=(95.66, 97.14)

**Qn.16** (a) (i) Line does not join the centres of the two clusters, even approximately.
(correct this in part (b)).
(ii) Eight points are below the line and only five above it. Also data itself is inherently curved. (correct this in part (b))

**Qn.17** (a) (i) $f(x)+3=(x-2)^2$
Write y for $f(x)+3$ and x for $(x-2)$ to obtain
$y = x^2$
(ii) Translating origin to (2,-3) to obtain the curve which is an downward pointing parabola with nose at (2,-3)
(iii) $(x, y) \mapsto (x+2, y-3)$
(b) (i) symmetrical letter V with apex at origin and internal angle of $90^0$.
(ii) h(x) = 3 + |x|
(c) points are given by the rightmost join of y=x+3 and f(x), and the leftmost join of y=3-x and f(x).
x-values of these points are given by the largest solution of $x^2-5x-2=0$ and the smallest solution of $x^2-3x-2=0$, which
are $\frac{1}{2}(5+\sqrt{33})=5.37$ and $\frac{1}{2}(3-\sqrt{17})=-0.56$
so points are (5.37, 8.37) and (-0.56, 3.56).

**Qn.18** (2002 candidates)
(a) Force diagram has 900N vertically downwards, T at $45^0$ upwards to right and R upwards to left at $10^0$ to vertical, all radiating outwards from the trolley.
(b) Triangle of forces has same forces chasing each other round a triangle.
(c) By sine rule $\frac{T}{\sin 10^o} = \frac{900}{\sin 125^o} = 1098.7$
so T= 190.8N
(d) Components are T $\cos 35^0$ and T $\sin 35^0$ that is 156.3 **i** + 109.4 **j** .

**Qn.19** (a) Reqd. Area=
$$\int_{-3}^{3}(9-x^2)dx = \left[9x - \frac{1}{3}x^3\right]_{-3}^{3}$$
= {(27-9) – (-27-9)} = 36

(b) If R is (x,0) PR = 3+x QR = $(9-x^2)$ so area A = ½(3+x) $(9-x^2)$ = ½ $(27 + 9x - 3x^2 - x^3)$
dA/dx = ½$(9 - 6x - 3x^2)$
= 0 when $9 - 6x - 3x^2 = 0$
i.e. $x^2 + 2x - 3 = (x + 3)(x - 1) = 0$
i.e. x =1 or x = -3
$d^2A/dx^2$ = ½(- 6 - 6x) which is <0 when x = 1, so corresponding value of A is a maximum.
Hence max. value of A is at x=1, namely ½.4.(9-1) = 16.

(c) Required area
$$= \{\int_{-3}^{1}(9-x^2)dx = \left[9x - \frac{1}{3}x^3\right]_{-3}^{1}\} - 16$$
= {(9 – 1/3) – (-27-9)}-16
= $26\frac{2}{3} - 16 = 10\frac{2}{3}$

**Qn.20** (a) age, gender, height.
(b)(i) $H_0 : \mu_A = \mu_B$
$H_1 : \mu_A \neq \mu_B$
where $H_0$ and $H_1$ are null and alternate hypotheses, and the mu's are the true mean weight losses of all patients who might (A) be given and (B) not be given the drug
(ii) $z = \frac{(5.01-1.36)}{\sqrt{\frac{2.72^2}{35}+\frac{2.15^2}{36}}}$ = 6.26

(iii) $z$ greatly exceeds 1.96 and so there is strong reason using a 5% level of significance to reject the null hypothesis. Also since the group A sample exceeds the group B sample mean, this means that there is good evidence that the drug is effective.