MST121 – 2001 Solutions

***** qns. 5 & 6 no longer in syllabus**

Qn.1 (a) 4.2, 0.84, 0.168, 0.0336
(b) Geometric Sequence

$x_n = 21(0.2)^{n-1}$ n=1,2,3..

(c) it tends to zero from above.

Qn.2 (a) $y = -2x + 22$

(b) 9.12

(c) (5½,2¾)

Qn.3 (a) $(x+2)^2+(y+3)^2=25$ radius=5, centre is (-2,-3)

(b) Solve $x^2+4x-12=0 \rightarrow (x-2)(x+6)=0 \Rightarrow$
so points are (2,0) and (-6,0)

(c) Answer to (b) eliminates all but B and C. Since centre (-2,-3) is in third quadrant, B must be the correct diagram.

Qn.4 (a) f(x) is straight line with negative gradient joining (0,3) and (1½,0)

g(x) is sine curve starting at y=4, initially turning downwards, and oscillating between 2 and 6.

(b) (i) *f* is one-to-one and so has an inverse.
g is not one-to-one since its values repeat for every increase of π in *x* and so has no inverse.

(b) (ii) $f^{-1}: \Re \rightarrow \Re$

$x \mapsto \frac{1}{2}(3-x)$

(n.b. formal function definition no longer in syllabus)

Qn.5 (a) 1,2,3,4,6,7 (b) gtst=2.1, least=1.9

Qn.6 (a) (i) $\dfrac{x+4}{\sin(\pi/2 - x)}$

not valid for *x*= odd multiples of $\pi/2$ because denominator is zero.

m1	m2	mem3	mem4
x			
x	x+4		
x	x+4	$(\pi/2 - x)$	
x	x+4	$\sin(\pi/2 - x)$	
x	x+4	$\sin(\pi/2 - x)$	$\dfrac{\sin(\pi/2 - x)}{x+4}$

(b) change last line to

memfour:=memtwo / memthree

Qn.7 (a) Sums are 3 . ½ . 4(5) and 3 . ½ . 20(21) which equal 30 and 630, so

$$\sum_{i=5}^{20}(3i-2) = \sum_{i=5}^{20}3i - \sum_{i=5}^{20}2$$

= 630 - 30 - 16x2 = 568

(b) When *n* is large

$$\frac{6n+4}{2-3n} \rightarrow \frac{6n}{-3n} \rightarrow -2$$

and $$\frac{6n+4}{2-3n^2} \rightarrow \frac{6n}{-3n^2} \rightarrow \frac{6}{-3n} \rightarrow 0$$

Qn.8 (a) *5x + 2y = 8*
4x - 3y =11

(b) det(M) = 5(-3)-(4.2) = -23

so $M^{-1} = \begin{pmatrix} 3/23 & 2/23 \\ 4/23 & -5/23 \end{pmatrix}$

soln.= $M^{-1}\begin{pmatrix} 8 \\ 11 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Qn.9 (a) $21t^2 - 2\exp(-2t)$
sum and const. mult. rules

(b) $\frac{1}{5}\sin(5x) + 2x\sqrt{x} + c$
sum and const. mult. rules

Qn.10 (a) $\dfrac{ds}{dx} = -\dfrac{1000\sqrt{3}}{x^2} + x\sqrt{3}$

$\dfrac{d^2s}{dx^2} = \dfrac{2000\sqrt{3}}{x^3} + \sqrt{3}$

$\dfrac{ds}{dx} = 0$ when $x^3 = 1000$, i.e. x=10

$\dfrac{d^2s}{dx^2} > 0$ when x=10 therefore min.

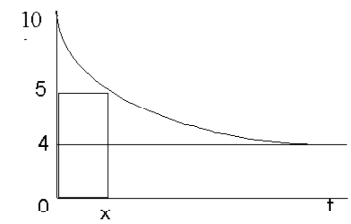
(b) when x=10 $s = 150\sqrt{3}$

(c) $vol = 250 = \frac{1}{4}\sqrt{3}(100)y \Rightarrow y = 10\sqrt{3}/3$

Qn.11 (a) Use $2as - v^2 = 2as_0 - v_0^2$
$2.a.80 = -576 \Rightarrow a = -3.6$, that is deceleration is 3.6 m/sec^2.

(b) Use $v = at + v_0$
$24 = 3.6t \Rightarrow t = 6\frac{2}{3}$ sec.

Qn.12 (a) with P axis in units of 1000:



(b) A=4000, B=6000

(c) 5000=4000+6000.exp(-0.05t)
ln(1/6)=-0.05t
t=ln(6)/0.05 = 35.8 yrs.

Qn.13 (a) Sketch is like a Normal curve but skewed slightly to the right.
(b) reqd. proportion is proportion of area under curve to right of vertical line through wt.=100
(c) Conf. Int.: limits are $73.8 \pm 1.96 \times \frac{14.1}{8}$,
i.e. (70.3,77.3)

Qn.14 (a) mean 28,median:24, LQ :15
UQ :34, Range:70, IQR : 19
(c) Right skewed because median < mean.

Qn.15 Accept H_0 ,i.e. there is no reason to believe that there is any significant difference between the diets with regard to their effect on weight loss.

Qn.16 (a) ln y – ln $x^{0.408}$ = 2.82
$y \big/ x^{0.408} = e^{2.82} = 16.8$ so k=16.8, b=0.408
and eqn .is y=$16.8x^{0.408}$
(b) $16.8(500^{0.408}) = 212$

Qn.17 2001 (a) (i) Solve -22 = 4r + d
-35=-22r + d
to give r= ½, d=-24
$x_n = \frac{1}{2}x_{n-1} - 24 \quad x_1 = 4$ (n=2,3,..)
(ii) $x_n = 52(\frac{1}{2})^{n-1} - 48$ (n=1,2,3,..) In the long term the sequence tends to –48.

(b) (i) y=2(x-1)+15 $\Rightarrow y = 2x + 13$ which is the eqn. of a straight line.
(ii) d=5(t-3)^2+45 so for min., t=3 at which time d^2=45.
(iii) $\sqrt{45} = 6.71$

Qn.17 2000 (a) (i) £1,422.50
(ii) $u_n = 1.015u_{n-1} \quad 100$, $u_0 = 1500$
(n=1,2,..)
(iii) Solve
$$0 = (1500 - \frac{100}{0.015})(1.015)^n + \frac{100}{0.015}$$
$$(1.015)^n = \frac{6666.67}{5167.67} = 1.2901$$
$$n = \frac{\ln 1.2901}{\ln 1.015} = 17.12$$
so loan is repaid in 18 months, following a final smaller monthly repayment.

Qn.18 (a) The equilibrium population.
(b) It will settle down to E=100 in an oscillatory manner because r=1.9 is between 1 and 2.
(c) $P_2 = 27\left[1 + 1.9(1 - \frac{27}{100})\right] = 64.449$
$$P_2 = 64.449\left[1 + 1.9(1 - \frac{64.449}{100})\right] = 107.98$$
One of these two values is above, and the other below the equilibrium value which is consistent with oscillation towards equilibrium value.
(d) plot is saw-tooth curve converging on 100 from either side.
(e) Unlikely since equilibrium level is likely to increase, and so in early years series is more nearly geometrical.

Qn.19 (a) $a(t) = -50\sin(2t)$,
(b) $x(t) = 12.5\sin(2t) + 32.5$
(c) Eliminate sin(2t): 4x = -a + 4(32.5)
$\Rightarrow a = 130 - 4x$
(d) at lowest point x is max at 45 ft. below platform.
max. accn is 50 when sin(2t)=-1
(e) $a = 10 \Rightarrow x = 30$. Max length is when x=45, which = 1½ of unstretched length.

Qn.20
(a) (i) (a) These are two independent events so probability(5,5) = 0.2 x 0.2 = 0.04.
(b) a "double" can happen in 5 ways, so probability = 5 x 0.04 = 0.2
(ii) (a) 0.8
(b) py(A chooses 1)=1/5 in which case py(B greater than A)= 4/5
Adding the probabilities for the four mutually exclusive possible ways in which B can choose a value greater than A's gives
$\frac{1}{5}(\frac{4}{5} + \frac{3}{5} + \frac{2}{5} + \frac{1}{5}) = 0.4$
(iii) The words "at least" indicate that you should calculate the probability of the "reverse" event, and then subtract this from 1
The reverse event is "same integer" for which py. is 0.2 so answer is 1 - $(0.2)^4$ = 0.9984.

(b) (i) $(1 \times \frac{4}{5} \times \frac{3}{5}) = 0.48$
(ii) 1-py(all three different) = 0.52