MST121 – 2000 Solutions

Qn.1 (a) 6.7, 6.4, 6.1, 5.8
(b) Arithmetic Progression
$x_n = 7 - 0.3n$ n=0,1,…
(c) x_n becomes indefinitely large and negative.

Qn.2 (a) 0.02, -0.002
(b) Geometric Progression
$x_{n+1} = -0.1x_n$ n=0,1,…
(c) x_n oscillates towards 0.

Qn.3(a) $(x-3)^2+(y-2)^2=16$
(b) $\sqrt{13}$
(c) Solve $(x-3)^2=12$, that is
$x^2-6x-3=0 \Rightarrow x = 3\pm\sqrt{9+3}$
=> x=6.464 or x=-0.464 so points are (6,464,0) and (-0.464,0).
(d) C is the only one consistent with result (c).

Qn.4 (a)1.2%
(b) (i) £1153.84 (ii) 15.384%
(c) $x \mapsto \ln(0.001x)/\ln(1.012)$

Qn.5 ***no longer in syllabus
(a) (i) 1,2,3
(ii)(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)
(b) D because it shades all points whose absolute x values are less than or equal to 5.

Qn.6 (a) No limit; for large i sequence is approx 5i/2.
(b) u_i=2/5i for large i since only first terms in num and denom matter and so limit is 0.

Qn.7 (a) about 1200, 4000
(b) (i) alternates between 4 values so r is about 2.5.
(ii) chaotic but bounded so r is between 2.6 and 3.
(iii) alternating between two specific values so r is between 2 and 2.3

Qn.8 (a) $\begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix}$, $\begin{pmatrix} 14 & 12 \\ -16 & -18 \end{pmatrix}$
(b) A= $\begin{pmatrix} 4 & 7 \\ 3 & 1 \end{pmatrix}$ b= $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$

A^{-1}= $\begin{pmatrix} -1/17 & 7/17 \\ 3/17 & -4/17 \end{pmatrix}$ soln.= $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Qn.9 (a) v = 1.8 – 2.4t
(b) 1.8 m/sec.
(c) s is max when v=ds/dt=0 ie. t=0.75 at which time s=0.675 m.
(d) a=dv/dt = -2.4 ms^{-2}

Qn.10 (a) $15e^{5x}-24x^3$;const multiple and sum rules
(b) $4\sqrt{t} + \frac{1}{8}\cos(8t) + c$;const multiple and sum rules

Qn.11 (a) 4,6 ; (6-x) and ln (x/4) are both >0 in (4<x<6).
$[\ldots]_4^6 = \{72(\ln 1.5 - 1) - 36\} - \{64(\ln 1 - 1) - 16\}$ = <u>1.193</u>

Qn.12 (a) rising exponential curve starting at (0,-10) and asymptotic to θ=20
(b) A=20, B=-30
(c) -10=-30.exp(-0.2t)
ln(1/3)=-0.2t
t=5.ln(3) = 5.493 (hrs.)

Qn.13 (a) 1/8 (b) ¼ (c) ¾
(d) 2 (prob. dist is

1	2	3	4 …	
½	¼	1/8	1/16	)

(e) First child is of one sex. The distribution of "first child of other sex" is then the same as that of "first girl" in (d) so mean no. of children=1+2 = 3.

Qn.14 (a) Left skewed - tail of distribution is to left.
(b) Sketch a smoothed curve ignoring the "bump" on the left corresponding to the 0-2 years column.
(c) Integrate area under this curve to right of line x=15, assuming that vertical scale is adjusted so total area under curve = 1.

Qn.15 (a) mean=81.6, sdev= 0.3
(b)(81.0,82.2)

Qn.16 (a) 169.6; 2.4cm. shorter
(b) 0.55x6 = 3.3cm.

Qn.17 (a) (i) A(-2,1) B(1,4)
(ii) x=t-2; t=x+2; so y=t+1=x+3
y=x+3 is eqn of str line,grad=1
(b) (i) $(t-2)^2+(t+1)^2=$
$t^2-4t+4 + t^2+2t+1 = 2t^2-2t+5$
$= 2(t-½)^2 + (5-½)$
(ii) max d^2 =4.5;
max d= $3/\sqrt{2}$ =2.12
(iii) C = (-1.5, 1.5)
(c) grad of OC = -1 perp. to AB

Qn.18 ***no longer in syllabus
(a) (i)

```
mem1  mem2  mem3        mem4
 x
 x    x+1
 x    x+1    x+3
 x    x+1   (x+3)²
 x    x+1   (x+3)²
(x+1)/(x+3)²
```

(ii) x not equal to -3

(b)(i)

```
              -
          /       \
       ÷            ÷
      / \          / \
     1   +        2   squ
        / \            |
       x   3           |
                       +
                     x   3
```

(ii) $\frac{1}{x+3}-\frac{2}{(x+3)^2} = \frac{(x+3)-2}{(x+3)^2}$

$= \frac{(x+1)}{(x+3)^2}$ = result of (i)

(iii)

```
mem1:=x
mem2:=mem1-4
mem2:=sin(mem2)
mem3:=mem1 x mem1
mem3:=mem3 + 3
mem3:=mem2 ÷ mem3
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Qn.19 (a) $\int_0^2 y\,dx=\left[6x-\frac{1}{2}x^3\right]_0^2=8$

(b)(i) min = area of ΔOPQ = 6
(ii) A(t) =
ΔQTQ'+ ΔPTP'+ rect.P'TQ'O
where P' and Q' are projections of T on y and x axes respectively.
= ½t(6-y)+ ½(2-t)y + ty
= 3t - ½ty + y - ½ty + ty
= 3t + y = 3t + (6 - 1.5t^2)
= $6+3t-\frac{3}{2}t^2$
(iii) dA/dt = 3-3t = 0 when t=1
d^2A/dt^2 =-3 <0
so max A = 6+3-1½ = 7½
(c) 6 < 7½ < 8

Qn.20 (a) Anne's *longer* sentences were longer, and her *overall* variation was greater.

(b)(i) $H_0 : \mu_A = \mu_B$
$H_1 : \mu_A \neq \mu_D$

where H_0 and H_1 are null and alternate hypotheses, and the μ 's are the absolute sentence length propensities of Anne and Charlotte.

(ii) $z=\frac{(36.7-32.9)}{\sqrt{\frac{33.0^2}{55}+\frac{23.0^2}{50}}}$ = 0.689

(iii) There is no reason to reject the null hypothesis, that is there is no evidence that Anne's inherent sentence length was different from Charlotte's.