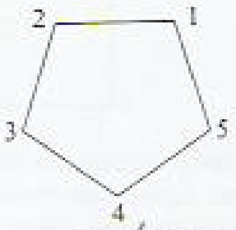
(54) This question concerns the symmetry group G of the regular pentagon.



1 2 3 5
2 1 2 1
3 5 1 2
4 4 5 3
5 3 4 4

$g=(12345)$ $h=(12)(35)$

Let $g \in G$ be the anti-clockwise rotation through $\frac{2\pi}{5}$ about the centre, and let $h \in G$ be the reflection in the vertical line through 4.

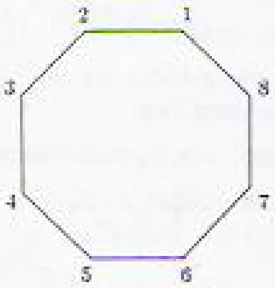
(i) Write g and h in cycle form, using the numbering of the locations of the vertices shown.

(ii) Determine the conjugate hgh^{-1} of g and state its order. (15432) $|hgh^{-1}|=5$

(iii) Give a brief reason why g and h are not conjugate. not same cycle structure

(iv) Write down an element of G other than h which is not conjugate to g.
$(13)(45)$

(55) This question is about symmetries of the regular octagon shown below. (90)



1 6 4 7
2 7 3 6
3 8 2 5
4 1 1 4
5 2 8 3
6 3 7 2
7 4 6 1
8 5 5 8

(a) Use the notation of the figure to express the following symmetries in cycle form:

(i) g: rotation through $3\pi/4$ anti-clockwise; (14725836) $g^{-1}(16385274)$

(ii) h: reflection in the diagonal through 1 and 5. $(28)(37)(46)$

(b) Find the conjugate ghg^{-1} and give a geometrical description of this symmetry. $(17)(26)(35)$

(c) Write down without proof two elements of the symmetry group that have the same cycle structure but are not conjugate as symmetries. [6]
$(12)(38)(47)(56)$ R_{π} $(15)(26)(37)(48)$

(56) This question concerns the symmetry group G of the regular hexagon.

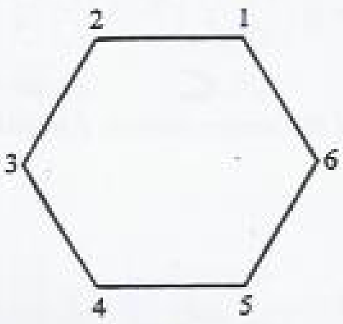


Figure 1

1 2 3 4 5 6
1 6 5 4 3 2
$(26)(35)$

Let $h \in G$ be rotation through $\frac{\pi}{3}$ anticlockwise. (123456)

(i) Write h as a permutation of the locations 1, 2, 3, 4, 5 and 6 of the vertices. [1]

(ii) Write down, as a permutation, an element g of order 2 in G such that $ghg^{-1} = h^{-1}$. [2]
$(1)(26)(35)(4)$

(iii) Explain why g is not conjugate to h in G. [1] different cycle structures

(iv) Let ϕ be a homomorphism from G to a group X.

Is the following statement true or false?

If h and h^{-1} are the elements of G referred to above, then there is an element x in X such that $x\phi(h)x^{-1} = \phi(h^{-1})$.

Briefly justify your answer. [2]

yes - conjugate in G
conjugate
in ϕ (82)