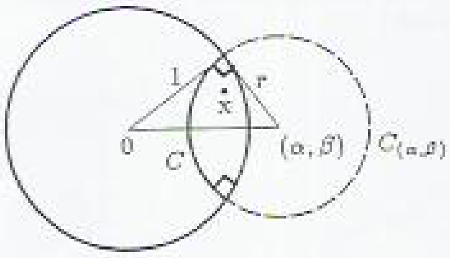
(12) This question concerns a *general reflection in the non-Euclidean disc* $\mathscr{D}$. Such a reflection is obtained by inversion in a given d-line. We assume here that this d-line is part of a circle $C_{(\alpha,\beta)}$ with centre (α,β) which meets $\mathscr{D}$ at right angles, as illustrated in the following diagram.



(a) Show that if the radius of the circle $C_{(\alpha,\beta)}$ is r, then

$$\alpha^2+\beta^2=r^2+1.$$ [1]

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(b) The image, $\phi(\mathrm{x})$, of a point x in $\mathscr{D}$ under inversion in $C_{(\alpha,\beta)}$ lies on the Euclidean line joining x to (α,β), and so has vector representation of the form

$$\phi(\mathrm{x})=\lambda\mathrm{x}+(1-\lambda)(\alpha,\beta),\quad \lambda\in\mathbb{R}.$$

$\phi(x)=\dfrac{\alpha\bar{x}-1}{\bar{x}-\bar{\alpha}}$

$=\dfrac{(\alpha+i\beta)\bar{x}-1}{\bar{x}-(\alpha+i\beta)}$

Determine the value of λ, and deduce that

$$\phi(\mathrm{x})=(\alpha,\beta)+\frac{\alpha^2+\beta^2-1}{\|\mathrm{x}-(\alpha,\beta)\|^2}(\mathrm{x}-(\alpha,\beta)).$$ [5]

(c) The point x_1 that maps to the origin $(0,0)$ under inversion in $C_{(\alpha,\beta)}$ must lie on the Euclidean line joining $(0,0)$ to (α,β).

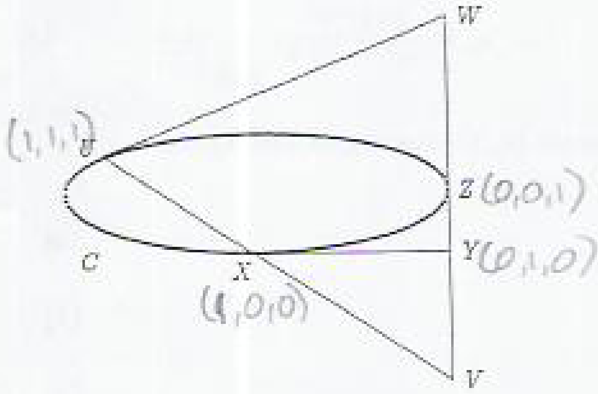
$(\bar{x}(\alpha+i\beta)-1)(\bar{x}+(\alpha+i\beta))$

$\bar{x}-$

Using this observation and the result of part (b), or otherwise, prove that

$$\mathrm{x}_1=\frac{(\alpha,\beta)}{\alpha^2+\beta^2}.$$ [4]

(13) This question concerns a general conic C in the projective plane.



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The tangents to C at X and Z meet at Y, and the tangent through the Point W of YZ meets C at U. The Line UX meets YZ at V.

(a) Taking XYZ as triangle of reference, write down the equation of the conic. [1]

(b) By making a suitable choice of coordinates for the Point U, find the equation of the tangent through U. Hence find the coordinates of the Point W. [4]

(c) Find the equation of the Line UX and hence the coordinates of the Point V. [2]

(d) Find the cross-ratio $(YZVW)$. [3]