(39) Sketch on separate diagrams the image under inversion in the unit circle $C = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ of: (94)

(A)

(a) the line $x = 2$;

(b) the line $x = 0$ (except the origin);

(c) the line $x = 1$.

(B)

(a) S_1: the line $y = 1$.

(b) S_2: the line $x = 3$.

(c) S_3: the circle with centre $(3,0)$ and radius 2.

(90)

(40) For each of the following Circles S_1, S_2 and S_3 sketch the image under inversion in the unit circle $C = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

(A)

(a) S_1: the line $y = 1$.

(b) S_2: the line $y = x$ (except the origin).

(c) $S_3 : \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 2\}$.

(91)

(B)

(a) S_1 : the circle with centre $(\frac{1}{2}, 0)$ and radius $\frac{1}{2}$; punctured at the origin. (93)

(b) S_2 : the line $x = \frac{1}{2}$;

(c) S_3 : the line $x = 2$.

You should indicate clearly on each diagram the Circle and its image.

(41) Find the affine transformation $t : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ such that (91)

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} 6 \\ 4 \end{pmatrix}.$$

Find the inverse of t, giving your answer in the form

$$t^{-1} : \mathbf{x} \longmapsto \mathbf{A}\mathbf{x} + \mathbf{b}.$$

[5]

(42) (a) Find an affine transformation t of the plane such that (89)

$$t : \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$t : \begin{pmatrix} 1 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$
$$t : \begin{pmatrix} 0 \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

[3]

(b) Find the inverse of t.

[2]

(43) (a) Find an affine transformation of the plane which maps $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 6 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ respectively.

[5] (80)

(b) Find the inverse of this map, in the form $t : \underline{x} \mapsto B\underline{x} + \underline{b}$.

(44)

(a) Find the equation of the projective conic that passes through the Points $[1,0,0]$, $[0,1,0]$, $[0,0,1]$, $[1,-1,1]$ and $[1,3,-2]$.

(b) Find the equation of the tangent at the Point $[1,3,-2]$ to the projective conic in part (a). (95)

[5]

(45) Find a projective transformation which maps the Points $[1,0,0]$, $[0,1,0]$, $[0,0,1]$, $[1,1,1]$ to the Points $[2,1,3]$, $[1,2,1]$, $[1,1,1]$, $[0,1,1]$ respectively. (89)

[5]

Find a projective transformation which maps the Points $[1,0,0]$, $[0,1,0]$, $[0,0,1]$, $[1,1,1]$ to the Points $[1,1,1]$, $[1,0,1]$, $[1,2,0]$, $[3,5,2]$ respectively. (88)

[5]