(62) This question is about the symmetry group G of the square shown below. (93)

Let $g \in G$ be the anti-clockwise rotation through $\pi/2$ about the centre, and let $h \in G$ be the reflection in the line through 1 and 3.

(a) Write g, g^2 and h in cycle form, using the numbering of the locations of the vertices as shown. $g=(1234)$ $g^2=(13)(24)$ $h=(24)$

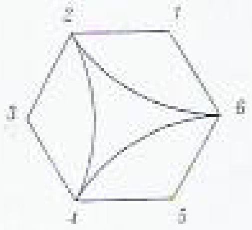
1 1 2 4
2 4 1 1
3 3 4 2
4 2 3 3

(b) Express the conjugate hgh^{-1} of g by h in cycle form and state its order. (1432)

(c) Give a brief reason why g^2 and h are not conjugate in G. [5]

different cycle struct.

(63) The plane figure below comprises a regular hexagon and some circular arcs of equal radius. (94)



(a) Write down the number of symmetries of the figure. 6

(b) List the symmetries of the figure in cycle notation, using the labelling of the vertex positions shown above.

(c) Write down a subgroup of the symmetry group of the figure that has order three. [5]

~~(e,(13)(24)56),(13)(24)~~

e,(135)(246),(153)(264)

(64) The sets G and H are defined as follows: (94)

$$G = \{1, 3, 7, 9, 11, 13, 17, 19\}; \quad H = \{1, 11\}.$$

The set G forms a group with the operation multiplication modulo 20. (You are NOT asked to prove this statement.)

(a) Show that H is a subgroup of G. closed, id, asso.

(b) Find the cosets of H in G. (3,13),(7,17),(9,19)

(c) Justify the statement that the quotient group G/H exists. cosets normal

(d) Identify the quotient group G/H as isomorphic to $\mathbb{Z}_2$, $\mathbb{Z}_4$, K_4, or $\mathbb{Z}_8$. [6]

K_4 ~~3 7 9~~

	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1