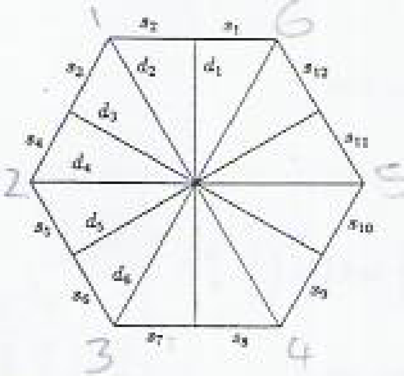
(3) The figure below shows a regular hexagon and its six axes of reflectional symmetry.



The set X consists of the twelve half-sides of the hexagon and the six axes of symmetry as shown on the figure. That is

(90/18) $$X = \{s_1, s_2, \ldots, s_{12}, d_1, d_2, \ldots, d_6\}.$$

The group $S(HEX)$, the group of symmetries of a regular hexagon, acts on X in the natural way: if $f \in S(HEX)$ and $x \in X$ then $f.x = f(x)$, the image of the line segment x under the symmetry f. You are NOT asked to prove that this is a group action.

(a) Write down the orbits of this action within X. $\frac{36}{12} = 3$ 12 [3]

(b) Describe geometrically the elements of:

(i) the stabilizer of s_7; e

(ii) the stabilizer of d_3; (e, reflection in d_3, reflection in d_6, rotation about π.

and determine the kernel of the action. Stab(⬡) = e [4]

(c) Explain how the Orbit Stabilizer Theorem applies to these results. [3]

Stab(e_x) = E

(4) The set $X = \{(1,0), (0,1), (-1,0), (0,-1)\}$ forms the vertices of a square drawn in the plane.

(a) Write down the eight matrices which represent the transformations of the plane that map this square to itself. [3]

The eight matrices in part (a) above form a group G (isomorphic to $S(\square)$). You are *NOT* asked to prove this. The group G acts on the plane $\mathbb{R}^2$ in the natural way.

(91/18) (b) For the action of G on the plane, find the orbit and the stabilizer of each of the following points.

(i) $(0,0)$

(ii) $(1,1)$

(iii) $(1,2)$ [5]

(c) Explain why, for any group action of G, the number of elements in an orbit can never be equal to 3. [2]

(5) Let G be the following group of matrices under matrix multiplication:

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 1/a \end{pmatrix} : a, b \in \mathbb{R}, a \neq 0 \right\}.$$

You are **not** *asked to* **verify** *that G is a group.*

A G-action on the plane $\mathbb{R}^2$ is defined by

(85/16) $$\begin{pmatrix} a & b \\ 0 & 1/a \end{pmatrix} \cdot (x, y) = (ax, ay).$$

(i) Show that this satisfies the two axioms for an action. [2]

(ii) Determine the stabiliser of $(2, 1) \in \mathbb{R}^2$. [2]

(iii) Determine the orbit of $(2, 1) \in \mathbb{R}^2$ under this action, giving a brief geometric description. [3]

(iv) Write down a *brief* description of the set of all orbits under this action. [3]