87 Use Stirling's Formula

(a) to estimate the following number to one significant figure:

$$\binom{160}{80}\sqrt{320\pi}\left(\frac{1}{2}\right)^{160};$$

(b) to determine a real number λ such that

$$\binom{3n}{n} \sim \lambda \frac{3^{3n}}{\sqrt{n}\,2^{2n}} \quad \text{as } n \longrightarrow \infty. \qquad [6]$$

88 Let $F: \mathbf{R} \longrightarrow \mathbf{R}$ and $G: \mathbf{R} \longrightarrow \mathbf{R}$ be functions such that

$$F(0) = 0, \quad G(0) = 1,$$

and for all x in $\mathbf{R}$,

$$DF(x) = e^x \sin x, \; DG(x) = e^x \cos x.$$

Using integration by parts, or otherwise, find F and G. [5]

89 The function f is defined by

$$f : [0,1] \longrightarrow \mathbf{R}$$

$$x \longmapsto \begin{cases} x, & x \in]0,1[; \\ \frac{1}{2}, & x = 0, 1. \end{cases}$$

(a) Sketch the graph of f.

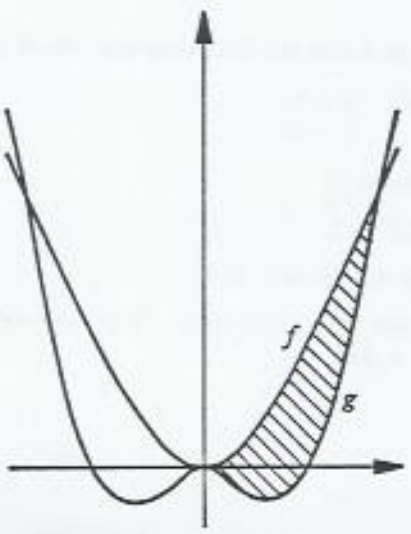
(b) Determine the values of the Riemann sums $L(f,P)$ and $U(f,P)$ for the partition $P = \{[0, 2/3], [2/3, 1]\}$. [4]

Let the functions $f: \mathbb{R} \longrightarrow \mathbb{R}$ and $g: \mathbb{R} \longrightarrow \mathbb{R}$ be given by

$$f(x) = x^2 \quad \text{and} \quad g(x) = x^4 - x^2 .$$

Find the area of the region between the graphs of f and g shown shaded in the diagram. [5]

90



91 The function f is defined on $[0,1]$ by

$$f(x) = \begin{cases} 1 - 2x, & 0 \le x < \frac{1}{2}, \\ 1 + x, & \frac{1}{2} \le x < 1, \\ 1, & x = 1. \end{cases}$$

(a) Sketch the graph of f.

(b) Determine the values of the Riemann sums $L(f,P)$ and $U(f,P)$ for the partition P of $[0,1]$ where

$$P = \left\{ \left[0, \tfrac{1}{2}\right], \left[\tfrac{1}{2}, \tfrac{2}{3}\right], \left[\tfrac{2}{3}, 1\right] \right\}. \qquad [5]$$