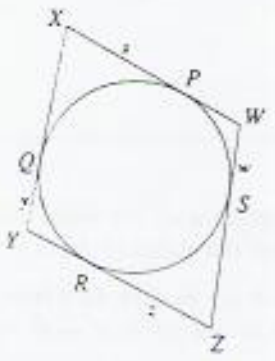
GEOMETRY

(1) (a) The sides of the parallelogram $WXYZ$ are tangents to a circle at the points P, Q, R and S, and the lengths of the segments WS, XP, YQ and ZR are w, x, y and z respectively, as shown in the following diagram.



(i) Show that

$$x + w = y + z$$

and

$$x + y = w + z.$$

(ii) Deduce from the two equations in part (i) that

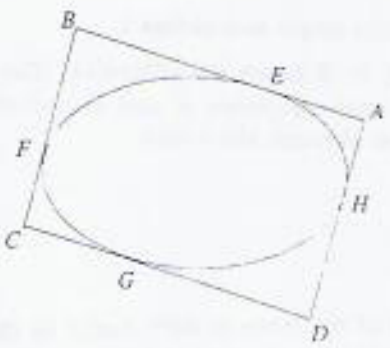
$$x = z$$

and

$$y = w.$$

(iii) Deduce from the two equations in part (ii) that $WXYZ$ is a parallelogram with four sides of equal length. [3]

(b) The sides of the parallelogram $ABCD$ are tangents to an ellipse at the points E, F, G and H, as shown in the following diagram.



(95)

By using an affine transformation that maps the ellipse to a circle, together with the results of part (a), prove that

$$\frac{AE}{EB} = \frac{CF}{FB}.$$ [7]