(G) 'Analysis'

(65) Determine whether each of the following sequences $\{a_n\}$ is convergent, stating the limit of the sequence (if it exists). You should name any result or test that you use.

(A)
(a) $a_n = \dfrac{3^n + 5n^2 - 3}{4^n + 3n + 1}, \quad n = 1, 2, \ldots$

(b) $a_n = \dfrac{n(-1)^n + 2}{3n + 3}, \quad n = 1, 2, \ldots$ (91)

(B)
(a) $a_n = \dfrac{n^2 + 4}{3n + 1}, \quad n = 1, 2, \ldots$ (93)

(b) $a_n = \dfrac{3^n - n! - 3}{3(n! + n + 1)}, \quad n = 1, 2, \ldots$

(66) Prove that each of the following sequences $\{a_n\}$ is convergent and determine its limit:

(i) $a_n = \dfrac{\cos n}{n^2}$; [3]

(ii) $a_n = \dfrac{5n^3 + 2n!}{n! + n^4}$. [3]

(87)

(67) In this question you should clearly name any rule for convergence that you use. You may assume that the sequence $\left\{\dfrac{1}{n}\right\}$ is convergent to zero, and that the function cos is continuous.

(i) Show that the sequence $\left\{1 + \cos\left(\dfrac{1}{n^2}\right)\right\}$ is convergent, and determine its limit.

(ii) Show that the sequence $\{s_n\}$, where

$$s_n = \frac{n^2\left(1 + \cos\left(\frac{1}{n^2}\right)\right) + n}{3n^2 - 5}$$

is convergent, and determine its limit. (86)

(68) In this question you should clearly name any rule for convergence which you use. You may assume that the sequence $\left\{\dfrac{1}{n}\right\}$ is convergent to 0, and that the function exp is continuous.

(i) Show that the sequence $\left\{\exp\left(\dfrac{1}{n^2}\right)\right\}$ is convergent, and determine its limit.

(ii) Show that the sequence $\{s_n\}$, where

$$s_n = \frac{n^2 \exp\left(\frac{1}{n^2}\right) + n}{2n^2 + n}$$

is convergent, and determine its limit. (85)

(69) (i) Find the limit of the sequence $\{s_n\}$, where

$$s_n = \frac{3n^2 + 4}{17n^2 + 6n}.$$ [2]

(ii) Use the Ratio Test to determine whether or not the infinite series

$$\sum_{n=1}^{\infty} \frac{3^n}{n^3} \text{ converges.}$$ [3] (81)

(70) (i) Find the limit of the sequence $\{s_n\}$ where

$$s_n = \frac{n^2 \exp(1/n) + n}{4n^2 + 2n - 1}.$$ [3]

(You may assume that the sequence $\{\exp(1/n)\}$ converges to 1.)

(ii) Using the fact that the series $\sum_{n=1}^{\infty} (\frac{1}{2})^n$ is convergent, show that the series $\sum_{n=1}^{\infty} a_n$, where

$$a_n = \left(\frac{|\cos n + \sin n|}{4}\right)^n, \text{ is convergent.}$$ [3] (82)