GROUP THEORY

(1) The sets $U = \left\{ \begin{pmatrix} a & c \\ 0 & d \end{pmatrix} : a, c, d, \in \mathbb{R}, ad \neq 0 \right\}$ and $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} : a, d \in \mathbb{R}, ad \neq 0 \right\}$ form groups under matrix multiplication. (You are NOT asked to prove these results.)

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The function $\phi : U \longrightarrow \mathbb{R}^*$ is defined by $\begin{pmatrix} a & c \\ 0 & d \end{pmatrix} \longmapsto ad$. (The operation on $\mathbb{R}^*$ is ordinary multiplication.)

(a) Show that ϕ is a homomorphism from U *onto* $\mathbb{R}^*$. [2]

(b) Find Ker(ϕ) and show that $\begin{pmatrix} 1 & 2 \\ 0 & 6 \end{pmatrix}$ and $\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ are in the same coset of Ker(ϕ). [2]

(c) Let $V = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$. Prove that V is a normal subgroup of U. [3]

(d) Construct a homomorphism from U onto G with kernel the set V defined in part (c), justifying your answer. [3]

(2) Consider the symmetry group S(PENT) of the regular pentagon shown below.

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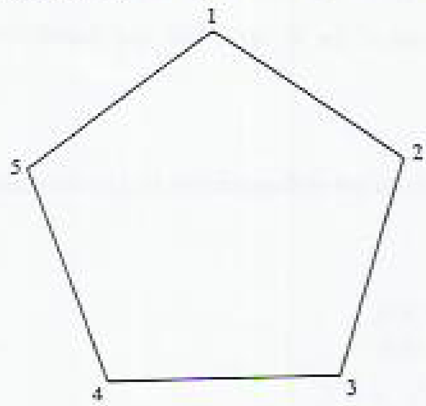
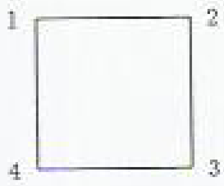


(i) Write down the elements of S(PENT) in *cycle* form, using the numbering of the locations of the vertices shown. [4]

(ii) Prove that the subgroup of direct symmetries S^+(PENT) is a normal subgroup of S(PENT). [4]

(iii) Give an example of a subgroup of S(PENT) which is not normal in S(PENT). Justify your answer. [2]

(3) Let G denote the symmetry group of the square, the locations of whose vertices are labelled as shown in the diagram below.



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In answering this question *you must use cycle notation.*

(a) List the conjugacy classes of G. For each class write down the order of any member. [3]

(b) Write down the three subgroups of G of order 4. (You are *NOT* asked to verify that they are subgroups.) [3]

(c) Explain why each of the subgroups in part (b) is normal. Write down (a group isomorphic to) the corresponding quotient group in each case. [3]

(d) State, without justification, whether the subgroup $D = \{e, (13)\}$ of G is a normal subgroup. [1]