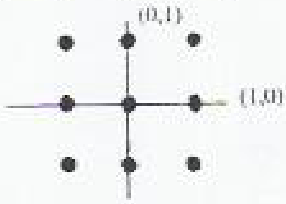
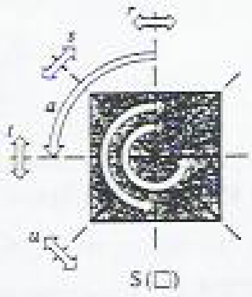
6

The set $X = \{(m,n) : m,n \in \{-1,0,1\}\}$ is shown below.



Let $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.



(93/18)

The group of matrices

$$G = \{\mathbf{I}, \mathbf{P}, \mathbf{P}^2, \mathbf{P}^3, \mathbf{Q}, \mathbf{PQ}, \mathbf{P}^2\mathbf{Q}, \mathbf{P}^3\mathbf{Q}\}$$

is isomorphic to

$$S(\square) = \{e, a, b, c, r, s, t, u\}.$$

(You are NOT asked to justify this result.)

An action of G on X is defined by $g \wedge (m,n) = (k,l)$ if $g\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} k \\ l \end{pmatrix}$, for any $g \in G$ and $(m,n) \in X$.

(a) By considering $\mathbf{P}_\wedge(m,n)$ and $\mathbf{Q}_\wedge(m,n)$ verify that axiom $GA1$ holds. [2]

In parts (b) and (c) below you may assume that G acts on X.

(b) Find the orbit and the stabilizer of each of the points:

(i) $(1,-1)$, {(

(ii) $(0,0)$, {0,0}

(iii) $(-1,0)$. [5]

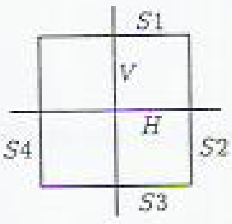
(c) Calculate $|\mathrm{Fix}(g)|$ for each $g \in G$ and use the Counting Theorem to show that there are exactly three orbits. [3]

7

Let G denote the symmetry group of the square:

$$G = \{e, \rho_{\pi/2}, \rho_\pi, \rho_{3\pi/2}, h, v, d_1, d_2\},$$

where the ρ's are the clockwise rotations through angles given by their subscripts, h and v are the reflections in the horizontal and vertical lines through the centre of the square, and d_1 and d_2 are the reflections in the two diagonals (through the top left and top right corners, respectively).



(89/18)

The sides are marked as shown in the above diagram and the vertical line V and the horizontal line H join the midpoints of the sides $S1$, $S3$ and $S2$, $S4$ respectively. Let $X = \{S1, S2, S3, S4, H, V\}$.

Let $\cdot$ denote the group action of G on X defined by

$$g \cdot x = g(x),$$

where $g \in G$ and $x \in X$.

(a) Write down all the orbits under the action of G on X. [2]

(b) Write down the kernel of the action. [1]

(c) Write down the stabilizer of each element of X, and find the intersection of all these stabilizers. [4]

(d) Prove that the stabilizer of $S1$ is conjugate in G to the stabilizer of $S2$. [3]