The following three questions are from part II of the 1994 paper. The separate parts (a), (b) would be suitable for the new Part I

92 (a) The function f is defined on $[0,1]$ by

$$f(x) = \begin{cases} 1, & x = 0, \\ 2x, & 0 < x < 1, \\ 0, & x = 1. \end{cases}$$

(i) Sketch the graph of f.

(ii) Determine the values of the Riemann sums $L(f,P)$ and $U(f,P)$ for the partition P of $[0,1]$ where

$$P = \left\{ \left[0, \tfrac{1}{3}\right], \left[\tfrac{1}{3}, \tfrac{1}{2}\right], \left[\tfrac{1}{2}, 1\right] \right\}.$$ [5]

92 (b) Show that

$$\int \frac{dx}{x(\log_e x)^{3/4}} = 4(\log_e x)^{1/4},$$

and hence determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\log_e n)^{3/4}}$$

converges or diverges. [5]

93 (a) Determine the Taylor polynomial $T_3(x)$ for the function $f(x) = (3+x)^{5/2}$ at 1. Show that $T_3(x)$ approximates $f(x)$ to within 10^{-2} on the interval $[1,2]$. [6]

93 (b) Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{4^n}{n}(x-1)^n.$$ [4]

94 (a) Determine the set of points at which the following function is continuous.

$$f(x) = \begin{cases} 1 + 2x + x^2, & x \leq 0, \\ e^x + x, & x > 0. \end{cases}$$ [6]

94 (b) Let p be the polynomial defined by

$$p(x) = x^5 + 2x^4 - 2x^3 - 2.$$

(i) Show that all real zeros of p lie in $]-3,3[$.

(ii) Prove that p has at least three real zeros. You may use the fact that $p(-3) = -29$ and $p(3) = 349$. [4]

95 The function f is defined by

$$f(x) = \begin{cases} 2 + 3x, & x < 1, \\ -3x^2 + 9x - 1, & x \geq 1. \end{cases}$$

Determine whether or not f is differentiable at the point 1. If it is differentiable, evaluate the derivative $f'(1)$.