E Geometry including affine maps'

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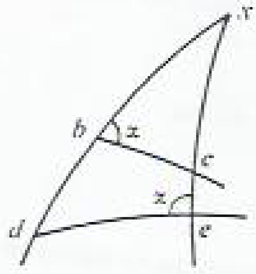
In the non-Euclidean plane, the points d and e lie on distinct lines through the point x. The point b, on the line segment xd, is distinct from both x and d. The point c, on the line segment xe, is such that $\angle xbc = \angle xed = \alpha$.



(i) Show that $\angle xde$ is less than $\angle xcb$.

(ii) Show that $\angle xde + \angle bce < \pi$.

(25)

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In the non-Euclidean plane, the points d and e lie on distinct lines through the point x and b is a point on the line segment xd distinct from both x and d. The point c on the line segment xe is such that $\angle xbc$ equals $\angle xde$.

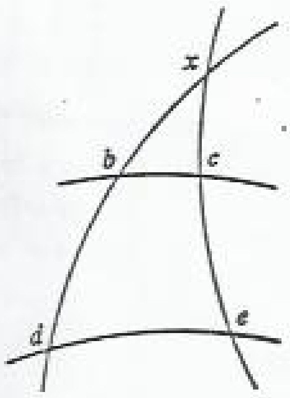

Figure 2

Show that the angles $\angle xcb$ and $\angle xed$ are not equal. [3] (81)

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(a) Find the Möbius transformation that maps the points $3i$, $-2-i$ and -4 to 0, 1 and ∞ respectively.

(b) Using the Möbius transformation that you found in part (a), show that $3i$, $-2-i$, -4 and 0 lie on a generalised circle. [6]

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(a) Find an affine transformation, t, of the plane which sends the triangle with vertices $\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ to the triangle with vertices $\begin{pmatrix}-2\\-1\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}1\\3\end{pmatrix}$, preserving the given order of the vertices.

(b) Find the inverse of t, giving your answer in the form

$$t^{-1} : \mathbf{x} \longmapsto \mathbf{A}\mathbf{x} + \mathbf{b}.$$

[5]