(11) (a) Determine at which points $x \in \mathbb{R}$ the following function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous:

(89/16)

$$f(x) = \begin{cases} 3 - 3x, & x < 0 \\ x^4 + 2x + 3, & x \geq 0 \end{cases}$$ [5]

(b) Prove that the following polynomial has at least three real zeros:

$$p(x) = x^5 + 3x^4 - x - 2.$$ [5]

(12) (i) **Determine at which points $x \in \mathbb{R}$ the following function $f : \mathbb{R} \to \mathbb{R}$ is continuous:**

$$f(x) = \begin{cases} 1 + x, & x < 0, \\ \cos x, & x \geq 0. \end{cases}$$ [5]

(88/16)

(ii) **Prove that the following function has at least one zero:**

$$f(x) = x^3 + 2x^2 + e^x \qquad (x \in \mathbb{R}).$$ [5]

(13) (i) Determine at which points $x \in \mathbb{R}$ the following function $f : \mathbb{R} \to \mathbb{R}$ is continuous:

(87/16)

$$f(x) = \begin{cases} 1 - x, & x < 0, \\ \sin x, & x \geq 0. \end{cases}$$ [5]

(ii) Prove that the following polynomial has at least three zeros

$$p(x) = x^5 - 2x^4 - x^3 + 2x + 1 \quad (x \in \mathbb{R}).$$ [5]

(14) (a) Is the following function continuous at 1?

$$f(x) = \begin{cases} e^x, & x \leq 1, \\ x + 1, & x > 1. \end{cases}$$

Justify your answer. [2]

(93/16)

(b) Determine all the points at which the following function is continuous.

$$f(x) = \begin{cases} 1 + 2x + x^4, & x \leq 0, \\ \cos^2 x + \sin x, & x > 0. \end{cases}$$ [5]

(c) Show that there is a value of x, $x \in [0, \pi]$, for which $x \cos^2 x = 1$. State clearly any results that you use. [3]