

D Matrices and Linear Transformations

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Find the matrix of the linear transformation

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y \\ x - y \end{pmatrix}$$

with respect to

- (i) the standard basis in both domain and codomain.
 (ii) the basis $\{(1, 2), (2, 2)\}$ in both domain and codomain.

[1]
 [3] (82)

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Find the matrix of the linear transformation

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (3y - x, 2x - 3y)$$

with respect to

- (i) the standard basis in both domain and codomain;
 (ii) the basis $\{(1, -2), (-1, 5)\}$ in the domain and the standard basis in the codomain;
 (iii) the basis $\{(1, -2), (-1, 5)\}$ in both domain and codomain.

(27)

[1]

[2]

[2]

23

Find the matrix of the linear transformation

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x - y \\ 2x + y \end{pmatrix}$$

with respect to

- (i) the standard basis in both domain and codomain
 (ii) the basis $\{(2, 1), (1, 2)\}$ in the domain, and the basis $\{(1, 1), (-2, -1)\}$ in the codomain.

(86)

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Find the matrix of the linear transformation

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (x + 2y, 3x + y)$$

with respect to

- (i) the standard basis in both domain and codomain;
 (ii) the basis $\{(-2, 1), (2, 3)\}$ in the domain and the standard basis in the codomain;
 (iii) the basis $\{(-2, 1), (2, 3)\}$ in both domain and codomain.

(82)

[1]

[2]

[2]

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The function f defined by

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (2x + 2y, x + 3y)$$

is a linear transformation. (You are NOT asked to verify this statement.)

- (a) Write down the matrix of f with respect to the standard basis in both the domain and the codomain.
 (b) Justify the statement that $\{(1, 2), (2, -1)\}$ is a basis for \mathbb{R}^2 .
 (c) Determine the matrix of f with respect to the basis $\{(1, 2), (2, -1)\}$ in both the domain and the codomain.

(92)

[4]

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The matrix $A = \begin{pmatrix} 3 & 3 \\ 5 & 1 \end{pmatrix}$ represents the linear transformation $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with respect to the standard basis in both the domain and codomain.

- (i) Determine the eigenvalues of f .
 (ii) Find a basis for each eigenspace of f .

(88)

[3]

[2]