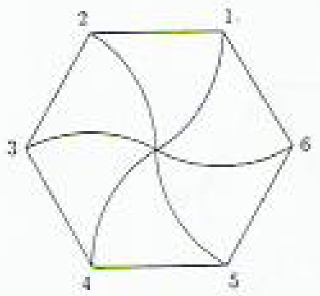
(F) 'Group Theory'

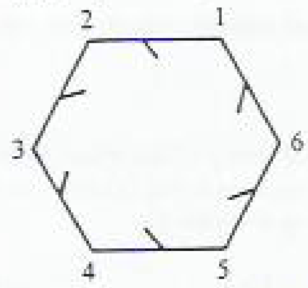
(47)



(a) Write down a group of permutations of $\{1, 2, 3, 4, 5, 6\}$ isomorphic to the symmetry group of the figure shown above. (e, $\frac{\pi}{3}$)

(b) Find all the cyclic subgroups of the group in part (a). (e, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$) [5]

(48) In this question, G is the symmetry group of the *plane* figure shown.



(an annotated regular hexagon)

(i) Write down the elements of G in bracket notation, and the order of G.

(ii) Write down a subgroup of G of order 2. (e, (14)(25)(36))

(iii) Write down, with brief reason, whether or not G is a cyclic group. Cyclic generated by (123456).

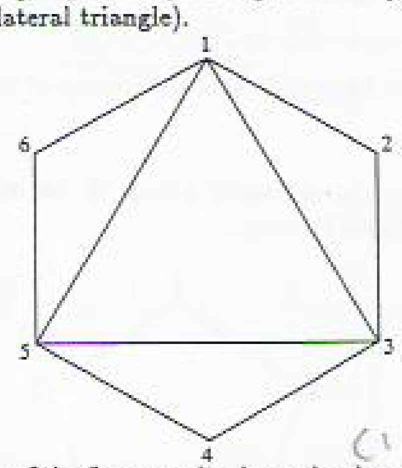
(iv) Prove that G is not (isomorphic to) a subgroup of the symmetry group of a square.

S(□) order 8. S(G) order 6

(86)

(82)

(49) Let G be the symmetry group of the plane figure below (which is a regular hexagon with an inscribed equilateral triangle).



△ order 6

⬡ order 12

(i) Using the labelling of the figure, write down the elements of G in any convenient notation, and state the order of G. G = (e, (153)(264), (135)(246), (26)(35), (15)(24), (13)(46)) |G| = 6 [3]

(ii) Write down a subgroup of G of order 3. (e, (153)(264), (135)(246)) [1]

(iii) Write down two subgroups of G which are isomorphic. [1]

(50) In this question G is the symmetry group of the square.

(i) How many elements are there in G? 8 [1]

(ii) By adding additional lines or marks to the square give a rough sketch of a figure whose symmetry group is a cyclic subgroup of G of order 4. [2]

(iii) Prove that G has no subgroup of order 3. [2] (82)