(10) This question is about the linear flow for which the velocity function is

$$V(x, y) = (3x - 2y, 4x - y).$$

(a) Write down:

(i) the matrix A of the flow;

(ii) the first order differential equations satisfied by the co-ordinate functions f and g of any flow function $\alpha = (f, g)$ for this flow;

(iii) a second order differential equation satisfied by both f and g. [3]

(b) Find the general solution of the differential equation in part (a) (iii). [3]

(c) Determine the flow function α for V that satisfies $\alpha(0) = (2, 1)$. [4]

(11) This question is about the linear flow for which the velocity function is

$$V(x, y) = (9x - 4y, \quad 24x - 11y).$$

(a) Write down

(i) the matrix A of the flow;

(ii) the first order differential equations satisfied by the co-ordinate functions f and g of any flow function $\alpha = (f, g)$ of this flow;

(iii) a second order differential equation satisfied by both f and g. [3]

(b) Find the general solution of the differential equation in part (a) (iii). [2]

(c) Determine the flow function α for V, and find the particular solution that satisfies $\alpha(0) = (1, 0)$. [5]

(12) Prove that the following functions are differentiable everywhere and determine the derived function of each.

(a) $f(x) = 2x^5 + 4x^3 - 3x^2$ [2]

(b) $g(x) = \begin{cases} x^4 \cos(2/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$ [5]

(c) $h(x) = \begin{cases} 2x^5 + 4x^3 - 3x^2, & x \leq 0, \\ x^4 \cos(2/x), & x > 0. \end{cases}$ [3]