(11) In this question V denotes the vector space of all polynomials in x of degree at most 2. That is

$$V = \{ax^2 + bx + c : a, b, c \in \mathbb{R}\}.$$

(You are NOT asked to prove that V is a vector space.)

(90/14)

(a) Determine whether or not each of the following sets is a subspace of V:

(i) $S = \{ax^2 + bx + c \in V : 2a + b = 0\}$;

(ii) $T = \{ax^2 + bx + c \in V : a = b, c = 1\}$. [4]

(b) Show that the function

$$t : V \longrightarrow \mathbb{R}^2$$
$$ax^2 + bx + c \longmapsto (2a + b, c).$$

is a linear transformation and determine the kernel of t in the form

$$\text{Ker}(t) = \{ax^2 + bx + c \in V : \text{ some conditions on } a, b, \text{ and } c\}.$$ [6]

(12) In this question, V denotes the vector space of all polynomials in x of degree at most 2, that is

$$V = \{p(x) : p(x) = ax^2 + bx + c, a, b, c \in \mathbb{R}\}.$$

(You are NOT asked to prove that V is a vector space.)

(93/14)

(a) Determine whether each of the following subsets of V is a subspace.

(i) $S = \{p(x) : p(x) = ax^2 - ax, a \in \mathbb{R}\}$

(ii) $T = \{p(x) : p(x) = ax^2 + bx + c, a, b, c \in \mathbb{R},\ a + b + c = 2\}$ [4]

(b) Show that the function

$$t : V \longrightarrow V$$
$$p(x) \longmapsto x\,p(0) - p(1)$$

is a linear transformation, and determine the kernel of t in the form $\text{Ker}(t) = \{p(x) : p(x) = ax^2 + bx + c \in V, \text{ some conditions on } a, b, c\}$. [6]

(13) In this question, we investigate the vector space V of all polynomials in x of degree 3 or less; that is, all polynomials of the form

$$p(x) = ax^3 + bx^2 + cx + d \qquad (a, b, c, d \in \mathbb{R}).$$

You are *not* asked to verify that V is a vector space.

(a) Determine whether the following two subsets S_1, S_2 of V are subspaces of V.

(i) $S_1 = \{p \in V : p(x) = x^2 - kx \text{ for some } k \in \mathbb{R}\}$.

(ii) $S_2 = \{p \in V : p(1) = p(-1)\}$. [5]

(89/14)

(b) Show that the function

$$t : V \longrightarrow V$$
$$p(x) \longmapsto p'(x) - x\,p(1)$$

?

is a linear transformation, and determine the kernel of t in the form

$$\{ax^3 + bx^2 + cx + d : \text{ some condition(s) on } a, b, c, d\}.$$ [5]

(14) In this question V denotes the vector space of functions given by $V = \{f : f(x) = ae^x + be^{-x} + c;\ a, b, c \in \mathbb{R}\}$. (You are NOT asked to prove that V is a vector space.)

(a) Determine whether each of the following subsets of V is a subspace of V.

(i) $S = \{f \in V : f(x) = a(e^x - e^{-x}), a \in \mathbb{R}\}$ ✓

(ii) $T = \{f \in V : f(0) = 2\}$ ✗ [5]

(94/14)

(b) Show that the function

$$t : V \longrightarrow \mathbb{R}^2$$
$$t : f \longmapsto (a - b, a + c), \quad \text{where } f(x) = ae^x + be^{-x} + c,$$

is a linear transformation, and determine the kernel of t in the form

$$\text{Ker}(t) = \{f \in V : f(x) = ae^x + be^{-x} + c;\ \text{some conditions on } a, b, c\}.$$ [5]