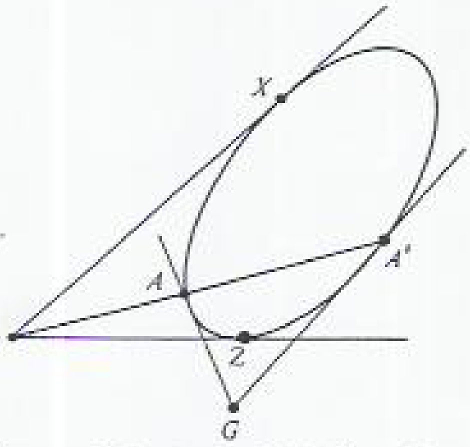
9 In the projective plane $X = [1,0,0]$, $Y = [0,1,0]$ and $Z = [0,0,1]$ and $y^2 = zx$ is the standard conic which touches the Lines XY and ZY at X and Z respectively.

(87/19)



(i) Show that any Point (except $[0,0,1]$) on the conic can be written as $[1, \alpha, \alpha^2]$. [2]

(ii) A and A' are two Points on the conic such that A, A' and Y are collinear. Find the coordinates of A' in terms of those of A. [2]

?

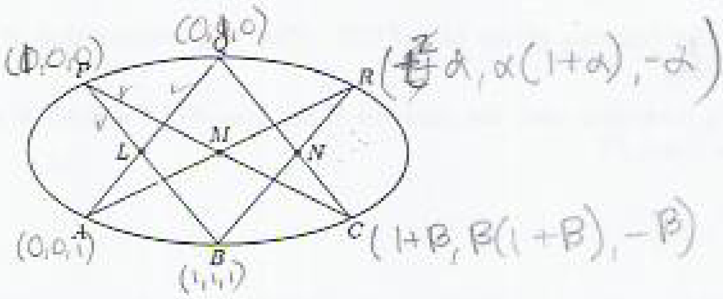
(iii) Verify that the Line

$$\alpha^2 x - 2\alpha y + z = 0$$

is tangent to the conic at $[1, \alpha, \alpha^2]$, for $\alpha \neq 0$. [4]

(iv) The tangents to the conic at A, A' meet at G. Prove that G lies on XZ. [2]

10 In this question we ask you to prove Pascal's theorem: if six points on a conic are joined as shown below then the three points L, M and N are collinear.



Take the equation of the conic to be

$$xy + yz + zx = 0.$$

(90/20)

(a) Show that the Points $X = [1,0,0]$, $Y = [0,1,0]$ and $Z = [0,0,1]$ all lie on the conic. [1]

(b) Show that any other Point on the conic can be written in the form

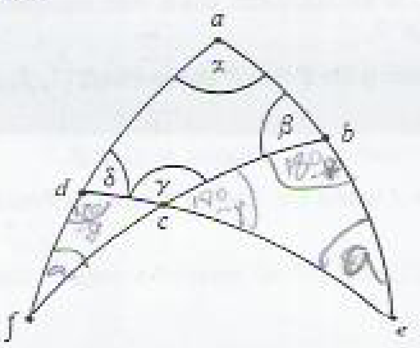
$$[1+\alpha, \alpha(1+\alpha), -\alpha] \quad \text{for } \alpha \neq 0, -1.$$ [3]

(c) Specify co-ordinates for the six Points A, B, C, P, Q and R on the conic and find the equations of the six Lines shown. [3]

(d) Hence find the Points L, M and N and prove that they are collinear. [3]

11 Let a, b, c, d be four distinct points in the non-Euclidean plane and suppose that the d-lines through a, b and d, c meet at e, whereas the d-lines through a, d and b, c meet at f. The angles $\alpha, \beta, \gamma, \delta$ are as specified in the diagram.

(87/20)



(i) Prove that

$$\beta + \gamma > \pi$$

and

$$\gamma + \delta > \pi.$$

(ii) Deduce that

$$\alpha < \gamma.$$