(5) Let XYZ denote the triangle of reference in $\mathbb{R}P^2$, with $X = [1, 0, 0]$, $Y = [0, 1, 0]$, $Z = [0, 0, 1]$. Let P be the Point $[1, 1, 1]$.

(i) Find the Points of intersection [4]

A of XP and YZ
B of YP and XZ
C of ZP and XY. [4]

(ii) Find the Points of intersection

D of BC and YZ
E of AC and XZ
F of AB and XY.

(iii) Show that D, E, F are collinear. (84) [2]

(6) Let XYZ be the triangle of reference in $\mathbb{R}\mathrm{P}^2$ and let $P = [\alpha, \beta, \gamma]$ be a general Point not lying on any side of the triangle. The Lines XP and YZ intersect in the Point A, YP and XZ intersect in B and ZP and XY intersect in C. (93)

(a) Find the equation of the Line XP and the co-ordinates of the Point A. Hence, or otherwise, write down the equations of the Lines YP and ZP and the coordinates of the Points B and C. [4]

(b) Find the Points of intersection :

F of BC and YZ,
G of AC and XZ,
H of AB and XY. [3]

(c) Find the equation of the Line FGH. (You may assume that F, G, H are collinear.) [2]

(d) If in a particular case, the equation of the Line FGH is $x + y + z = 0$, what is the Point P? [1]

(7) This question concerns the standard conic in the projective plane, with equation $y^2 = xz$.

(i) Verify that the equation of the tangent to the conic

$$y^2 = xz$$ (80)

at the Point $P = [1, \alpha, \alpha^2]$ is

$$2y\alpha = \alpha^2 x + z .$$

(ii) Write down the equation of the Line ℓ joining the Point $Y = [0, 1, 0]$ to the Point $P = [1, \alpha, \alpha^2]$. $\alpha^2 x + z = 0$

(iii) Find the other point Q of intersection of the Line ℓ and the conic.

(iv) Write down the equation of the tangent to the conic at Q.

(v) Deduce that the tangents to the conic at P and Q meet on the Line $y = 0$.

(8) In the projective plane, XYZ is the triangle of reference (with $X = [1, 0, 0]$, $Y = [0, 1, 0]$ and $Z = [0, 0, 1]$) and C is the standard conic with equation $y^2 = xz$. Let P be the Point $[1, \alpha, \alpha^2]$, where $\alpha \neq 0$. (81)

(i) Find the equation of the Line joining the Point P to the Point $[1, \beta, \beta^2]$ where $\beta \neq \alpha$. [2]

(ii) Find the equation of the Line ℓ which is the tangent to C at P. [3]

(iii) Find the Point Q where the Line ℓ meets the Line XY and the Point R where ℓ meets the Line YZ. [3]

(iv) Prove that the three Lines XR, ZQ and YP all meet at a single Point. [2]