(71) Determine whether each of the following series is convergent. (You should name any result or Test that you use.) (89)

(a) $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$. [3]

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{4n-3}$. [3]

(72) Determine whether or not each of the following series is convergent:

(a) $\sum_{n=1}^{\infty} \frac{3(n!)^2}{(2n)!}$;

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+2n^{2/3}}$. [6]

(73) Determine whether each of the following series is convergent: (88)

(A) (i) $\sum_{n=1}^{\infty} \frac{3n+1}{n^3}$;

(ii) $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$. [6]

(B) (a) $\sum_{n=1}^{\infty} \frac{n^2 3^n}{n!}$ (94)

(b) $\sum_{n=1}^{\infty} \frac{n^3}{2n+n^3}$ [5]

(74) (A) Prove that (88)

$$\lim_{x \to 0} \frac{x - \sin x}{1 - \cos x}$$

exists, and determine its value.

(B) Prove that (93)

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2 \cos x}$$

exists and determine its value.

(75) Prove that (87)

$$\lim_{x \to \pi/2} \frac{\cos 3x + \cos 5x}{\sin 3x + e^{\cos x}}$$

exists, and determine its value. [5]

(76) Prove that (90)

$$\lim_{x \to 0} \frac{\sin(2x) - 2x + 3x^2}{\sin(x^2)}$$

exists, and determine its value. [6]

(77) Given that the identity function and the exponential function are continuous at all points of R, show, using the rules for continuous functions, that the function $f: x \longmapsto x + e^{x^2}$ is continuous at all points of R. Clearly name each rule whenever you use it. (80)

[4]

(78) Use the Increasing-Decreasing Theorem to prove that

$$x^{3/4} \le \tfrac{3}{4}x + \tfrac{1}{4}, \qquad \text{for } x \in [0,1].$$ [5] (94)

(79) Show, by using the rules for differentiation, that the function $f\colon \mathrm{R} \longrightarrow \mathrm{R}$ given by

$$x \longmapsto \begin{cases} x \log_e x, & x \geqslant 1 \\ \tfrac{1}{2}(x^2 - 1), & x \leqslant 1 \end{cases}$$

is differentiable at 1. (*Each rule you use should be clearly identified. You may assume that the identity function, the* $\log_e$ *function and constant functions are differentiable at* 1.) [5] (82)