27 The matrix $A = \begin{pmatrix} -12 & 8 \\ -15 & 10 \end{pmatrix}$ represents the linear transformation $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ with respect to the standard basis in both domain and codomain. (91)

(a) Determine the eigenvalues of f.

(b) Find a basis for each eigenspace of f.

(c) Write down a transition matrix P such that the matrix $P^{-1}AP$ is diagonal; write down the corresponding diagonal matrix. [6]

28A (i) Find the row-reduced form of the matrix

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$ [4]

(ii) Hence write down whether the simultaneous equations

$$\begin{aligned} x + 2y - z &= 3 \\ 2x + y + z &= 3 \\ y + z &= 3 \end{aligned}$$

have a *unique* solution in x, y, z. Briefly justify your answer. [1] (92)

28B (a) Determine the row-reduced form of the matrix

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 3 \\ 3 & -1 & -11 \end{pmatrix}.$$ (91)

(b) Hence, or otherwise, find the solution set of the equations

$$\begin{aligned} 2x + y + z &= 0 \\ x + y + 3z &= 0 \\ 3x - y - 11z &= 0. \end{aligned}$$ [5]

28C (a) Determine the row-reduced form of the matrix

$$\begin{pmatrix} 1 & 3 & 7 \\ 3 & 2 & 7 \\ -1 & 1 & 1 \end{pmatrix}.$$ (93)

(b) Hence, or otherwise, find the solution set of the equations

$$\begin{aligned} x + 3y + 7z &= 0, \\ 3x + 2y + 7z &= 0, \\ -x + y + z &= 0. \end{aligned}$$ [5]

29 (i) Find the row-reduced form of the matrix

$$\begin{pmatrix} 1 & 4 & 3 \\ 2 & 10 & 15 \\ 1 & 4 & 6 \end{pmatrix}.$$

(ii) Hence write down whether the simultaneous equations

$$\begin{aligned} x + 4y + 3z &= 11 \\ 2x + 10y + 15z &= 4 \\ x + 4y + 6z &= 8 \end{aligned}$$ (84)

have a unique solution in x, y, z. Briefly justify your answer. (*There is no need to solve the equations.*)

30 (i) Find the row-reduced form of the matrix

$$\begin{pmatrix} 1 & 1 & -3 \\ 2 & 3 & -6 \\ 1 & 2 & 3 \end{pmatrix}$$

(ii) Hence find the solution set of the equations

$$\begin{aligned} x + y - 3z &= 0 \\ 2x + 3y - 6z &= 0 \\ x + 2y + 3z &= 0. \end{aligned}$$ (86)

31 (i) Find the row-reduced form of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$. [4] (80)

(ii) Hence write down whether there are any values of a, b, c for which the simultaneous equations

$$\begin{aligned} 2x + y + z &= a \\ x + y &= b \\ 2x - z &= c \end{aligned}$$

have a unique solution in x, y, z. [1] (80)