80 (i) Write down the derivative Dg of the function

$$g : \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto \sin^2(e^{3x}).$$

$B1-6\sin(e^{3x})\cos(e^{3x})$

(ii) Show that the function

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto \begin{cases} 1 + x & \text{if } x < 0 \\ 1 + x + x^2 & \text{if } x \geqq 0 \end{cases}$$

is differentiable at every point of $\mathbb{R}$. State whether the function Df is differentiable at 0. (A rough sketch of the graph of Df near the origin should help.) (85)

81 (i) Write down the derivative Dg of the function

$$g : \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto e^{\cos^2 x}.$$

$+2\sin x\cos x e^{\cos^2 x}$

(ii) Show that the function

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto \begin{cases} -x^3 & \text{if } x < 0 \\ x^2 & \text{if } x \geqq 0 \end{cases}$$

is differentiable at every point of $\mathbb{R}$. State whether the function Df is differentiable at 0. (A rough sketch of the graph of Df near the origin should help.) (86)

82 (a) Determine the tangent approximation to the function (89) //(95)

$$f(x) = 3 + 2x + x^5 - 2e^{-x+1}$$

at the point 1.

(b) Determine the interval of convergence of the following power series:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n} x^n.$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{5n\,2^n}(x+2)^n.$

(88)

83 (i) Calculate the Taylor polynomial $T_2(x)$ for the function

$$f(x) = \log_e(1 + \tfrac{1}{2}x)$$ [3]

at 0.

(ii) Show that $T_2(x)$ approximates $f(x)$ to within 5×10^{-2} on the interval [0,1]. [3]

84 (i) Write down the Taylor series at 0 for the function

$$f(x) = \log_e(1 + 2x^2) \quad (x \in \mathbb{R}),$$

indicating the general term. State the radius of convergence of the series. [3]

(ii) Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^n.$$ [3]

(87)

85 (a) Determine the barrier lines (if any) for the flow with velocity function (91)

$$V(x, y) = (x + 2y, 2x - 2y)$$

and the direction of the flow on each barrier line.

(b) Hence draw a rough sketch of the flow. [5]

86 (a) Determine the barrier lines for the flow with velocity function (89)

$$V(x, y) = (2x + 2y, 6x + y),$$

and the direction of the flow on each. [4]

(b) Hence draw a rough sketch of the flow. [1]