(c) The vector $(a, b, -a-b)$ lies in S and is perpendicular to $(1, 2, -3)$ (which belongs to S) if

$$(1, 2, -3) \cdot (a, b, -a-b) = 0; \qquad \text{1M}$$

that is,

$$a + 2b + 3a + 3b = 0,$$

or

$$4a + 5b = 0,$$

for which one solution is $a = 5$, $b = -4$. This gives

$$(a, b, -a-b) = (5, -4, -1). \qquad \text{1A}$$

Hence an orthogonal basis for S is

$$\{(1, 2, -3), (5, -4, -1)\}.$$

(d) We have to solve

$$(4, -1, -3) = \alpha(1, 2, -3) + \beta(5, -4, -1) \qquad (*)$$

for α, β. Since $(1, 2, -3)$ and $(5, -4, -1)$ are orthogonal, we have 1M

$$\alpha = \frac{(4, -1, -3) \cdot (1, 2, -3)}{(1, 2, -3) \cdot (1, 2, -3)} = \frac{11}{14} \qquad \tfrac{1}{2}\text{A}$$

and

$$\beta = \frac{(4, -1, -3) \cdot (5, -4, -1)}{(5, -4, -1) \cdot (5, -4, -1)} = \frac{27}{42} = \frac{9}{14}. \qquad \tfrac{1}{2}\text{A}$$

Thus

$$(4, -1, -3) = \tfrac{11}{14}(1, 2, -3) + \tfrac{9}{14}(5, -4, -1).$$

Alternatively, solve $(*)$ for α, β.

Question 17

(a) We prove that f is continuous at 0, using the Glue Rule. Let g and h be the functions

$$g(x) = x^3 \qquad (x \in \mathbb{R}),$$
$$h(x) = -x^2 \qquad (x \in \mathbb{R}), \qquad \tfrac{1}{2}\text{M}$$

and let $I = \mathbb{R}$. Then the functions f, g, h are defined on I, and

1. $f(x) = g(x)$ for $x < 0$,
   $f(x) = h(x)$ for $x > 0$; $\tfrac{1}{2}$A
2. $g(0) = h(0) = 0 = f(0)$; $\tfrac{1}{2}$A
3. the functions g, h are both continuous at 0 since they are polynomials. $\tfrac{1}{2}$A

Hence f is continuous at 0, by the Glue Rule. 1A ($\tfrac{1}{2}$ for quoting the correct rule)

(b) We prove that f is discontinuous at 0, by finding a sequence $\{x_n\}$ such that $x_n \to 0$ but

$$f(x_n) \not\to f(0) = 0. \qquad (*) \qquad \tfrac{1}{2}\text{M}$$

We can take any sequence $\{x_n\}$ which tends to 0 from the right. For example, take

$$x_n = \frac{1}{n}, \quad n = 1, 2, \ldots. \qquad \tfrac{1}{2}\text{A}$$

Then $x_n \to 0$ but

$$f(x_n) = f\left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)^{-2} = n^2. \qquad \text{1A}$$

Since $n^2 \to \infty$ as $n \to \infty$, we have verified $(*)$; and so f is discontinuous at 0. 1A