(b) When $t = 2$, $x = 4$ and $y = 9$. $\frac{1}{2}$A

The slope of the tangent is

$$\frac{dy}{dx} = \frac{dy}{dt} \bigg/ \frac{dx}{dt}.$$

Now $y = 2t^2 + 1$, so $\frac{dy}{dt} = 4t$; $x = 2t$, so $\frac{dx}{dt} = 2$.

So at $t = 2$, $\frac{dy}{dt} = 8$, and hence the gradient of the tangent to the curve at the point $t = 2$ is $8/2 = 4$. $\frac{1}{2}$A, $\frac{1}{2}$M

Hence the equation of the tangent is

$$y - 9 = 4(x - 4)$$

or $\quad y = 4x - 7$. $\frac{1}{2}$A

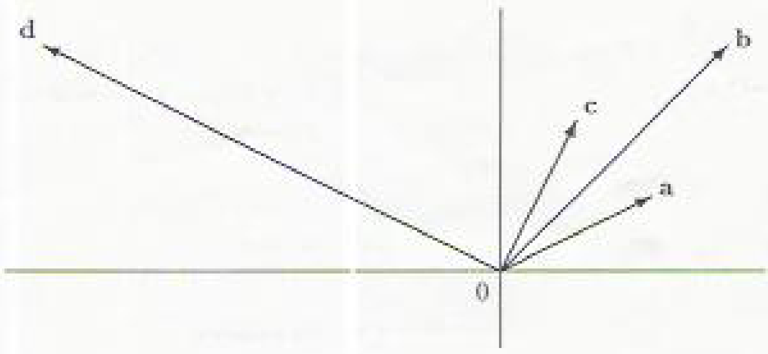
Question 2

(a) The position vectors $\mathbf{c}$ and $\mathbf{d}$ are given by

$$\mathbf{c} = (3,3) - (2,1) = (1,2),$$ 1A

$$\mathbf{d} = 9(1,2) - 5(3,3) = (-6,3).$$ 1A

1A



(b) $\|\mathbf{d}\| = \sqrt{(-6)^2 + 3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$. $\frac{1}{2}$A

(c) We consider the dot product $\frac{1}{2}$M We look for a zero dot product.

$$\mathbf{c} \cdot \mathbf{d} = (1,2) \cdot (-6,3)$$

$$= -6 + 6 = 0.$$ $\frac{1}{2}$A

Hence $\mathbf{c}$ and $\mathbf{d}$ are perpendicular because their dot product is zero. $\frac{1}{2}$M

Question 3

(a) The standard basis is $\{(1,0),(0,1)\}$.

$$f : \begin{pmatrix} 1 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 0-2 \\ 1+0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix},$$

$$f : \begin{pmatrix} 0 \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} 1-0 \\ 0+3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

Hence the matrix is $\begin{pmatrix} -2 & 1 \\ 1 & 3 \end{pmatrix}$. 1A

(b) $$f : \begin{pmatrix} 2 \\ -1 \end{pmatrix} \longmapsto \begin{pmatrix} -1-4 \\ 2-3 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix},$$ 1M

$$f : \begin{pmatrix} 3 \\ 2 \end{pmatrix} \longmapsto \begin{pmatrix} 2-6 \\ 3+6 \end{pmatrix} = \begin{pmatrix} -4 \\ 9 \end{pmatrix}.$$

Hence the matrix is $\begin{pmatrix} -5 & -4 \\ -1 & 9 \end{pmatrix}$. 1A