(b) Since $\{n^3/2^n\}$ is a basic null sequence with positive terms, $\{2^n/n^3\}$ tends to infinity, by the Reciprocal Rule. 1A $\frac{1}{2}$M

Hence $\{2^n/n^3\}$ is not a null sequence, and so

$$\sum_{n=1}^{\infty} \frac{2^n}{n^3} \quad \text{is divergent,}$$ 1A

by the Non-null Test. $\frac{1}{2}$M

Question 8

(a) $f \circ g = (235) \circ (13)(24)$ — 1 mark penalty for 2-line form

$= (15243);$ $\frac{1}{2}$A

$f \circ g \circ h = (f \circ g) \circ h$

$= (15243) \circ (13542)$

$= (1)(253)(4) = (253);$ $\frac{1}{2}$M, $\frac{1}{2}$A

$h^{-1} = (24531) = (12453);$ $\frac{1}{2}$A

$f \circ g \circ f^{-1} = (15)(34).$ $\frac{1}{2}$M, $\frac{1}{2}$A (Remember that $f \circ g \circ f^{-1} = (f(1)f(3))(f(2)f(4))$.)

(b) Using the method illustrated in the proof of Cayley's Theorem, we obtain one such group to be

$$\left\{ \begin{pmatrix} p & q & r & s \\ s & r & q & p \end{pmatrix}, \begin{pmatrix} p & q & r & s \\ r & s & p & q \end{pmatrix}, \begin{pmatrix} p & q & r & s \\ q & p & s & r \end{pmatrix}, \begin{pmatrix} p & q & r & s \\ p & q & r & s \end{pmatrix} = e \right\}$$ 2A

(No reasons required.)

or

$\{e, (ps)(qr), (pr)(qs), (pq)(rs)\}.$

Question 9

(a) Let x, y be elements of $\mathbb{R}^*$. Then

$$\begin{aligned} \theta(xy) &= (xy)^2 \\ &= x^2y^2 \qquad (\text{since } x, y \in \mathbb{R}) \\ &= \theta(x)\theta(y). \end{aligned}$$

Hence θ is a homomorphism. 1M, 1A

(b) The identity of $\mathbb{R}^*$ is 1, and so

$\text{Ker}(\theta) = \{x \in \mathbb{R}^* : \theta(x) = 1\}$ 1M

$= \{x \in \mathbb{R}^* : x^2 = 1\}$

$= \{1, -1\}.$ 1A

Question 10

(a) $\{C1, C2, C3, C4\}$, $\{C\}$, $\{S1, S3\}$, $\{S2, S4\}$, $\{D1, D2\}$. 3A

(b) The stabilizer of $C1$ is $\{e\}$. $\frac{1}{2}$A

The stabilizer of C is $S(\square)$. $\frac{1}{2}$A

The stabilizer of $S1$ is $\{e, r\}$. 1A