Question 13

Show that

$$\int \frac{dx}{x(\log_e x)^{4/3}} = -3(\log_e x)^{-1/3},$$

and deduce whether or not the following series is convergent:

$$\sum_{n=2}^{\infty} \frac{1}{n(\log_e n)^{4/3}}. \qquad [5]$$

Question 14

(a) Calculate the Taylor polynomial $T_3(x)$ for the function $f(x) = 1/x$ at 2.

(b) Show that $T_3(x)$ approximates $f(x)$ to within 10^{-2} on the interval $\left[2, \frac{5}{2}\right]$. [5]