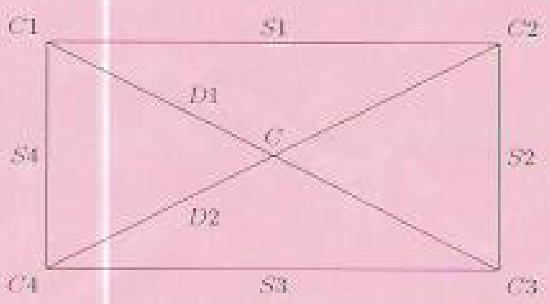
Question 10

The rectangle shown below has corners $C1$, $C2$, $C3$, $C4$, sides $S1$, $S2$, $S3$, $S4$, centre C and diagonals $D1$, $D2$. (These locations are fixed in the plane.)



The symmetry group of the rectangle is $S(\square) = \{e, a, r, s\}$, where e is the identity, a is rotation through π about C, r is reflection in the vertical line of symmetry, and s is reflection in the horizontal line of symmetry.

The group $S(\square)$ defines a group action on the set

$$X = \{C1, C2, C3, C4, C, S1, S2, S3, S4, D1, D2\}$$

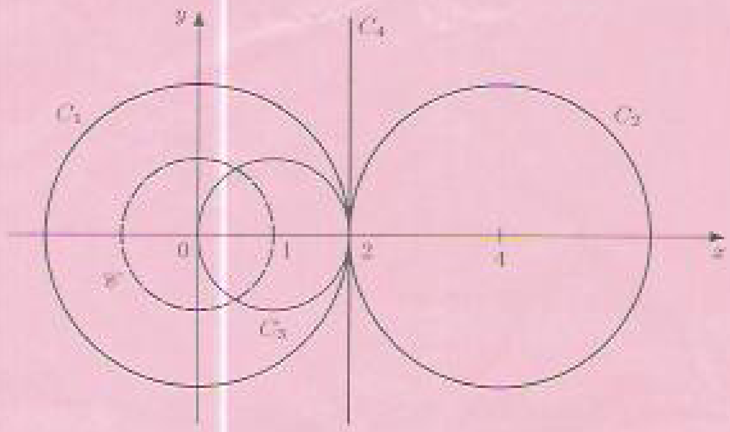
as follows: for $g \in G$ and $x \in X$, $g \wedge x$ is the position to which x is moved by g. (You are **not** asked to prove that this is a group action.)

(a) Write down all the orbits under the action of $S(\square)$ on X.

(b) Write down the stabilizers of $C1$, C and $S1$. [5]

Question 11

Let C_1 and C_2 be circles of radius 2 with centres at $(0, 0)$ and $(4, 0)$ respectively, let C_3 be a circle of radius 1 with centre at $(1, 0)$, and let C_4 be the line $x = 2$.



Draw a sketch showing the images of C_1, C_2, C_3 and C_4 under inversion in the unit circle $\mathscr{C}$. Mark clearly which image is which. (There is no need to determine the equations of either the given circles or their images.) [5]

Question 12

Find a projective transformation t which maps the Points $[1, 0, 0]$, $[0, 1, 0]$, $[0, 0, 1]$, $[1, 1, 1]$ to the Points $[0, 0, 1]$, $[1, 0, 0]$, $[1, 1, 1]$, $[3, 2, 1]$, respectively. [5]