PART I

(i) *You should attempt as many questions as you can in this part.*

(ii) *Write your answers in the answer book provided, beginning each question on a new page.*

(iii) *Questions in this part do not necessarily carry equal marks. The mark allocation is indicated for each question.*

Question 1

(a) Determine whether the conic defined by the parametric equations

$$x = 2t, \quad y = 2t^2 + 1$$

is an ellipse, a parabola or a hyperbola.

Sketch this curve, indicating clearly the points that correspond to the values $t = 0$, $t = 1$ and $t = -2$ of the parameter t.

(b) Find the equation of the tangent to the conic at the point where $t = 2$. [5]

Question 2

Let $\mathbf{a}$ and $\mathbf{b}$ be the position vectors $(2, 1)$ and $(3, 3)$, respectively.

(a) Determine the position vectors $\mathbf{c} = \mathbf{b} - \mathbf{a}$ and $\mathbf{d} = 9\mathbf{c} - 5\mathbf{b}$.

Draw a rough sketch of the plane showing the points with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$.

(b) Determine the length of $\mathbf{d}$.

(c) Show that the vectors $\mathbf{c}$ and $\mathbf{d}$ are perpendicular to each other. [5]

Question 3

Find the matrix of the linear transformation

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$(x, y) \longmapsto (y - 2x, x + 3y)$$

with respect to

(a) the standard basis in both domain and codomain;

(b) the basis $\{(2, -1), (3, 2)\}$ in the domain and the standard basis in the codomain;

(c) the basis $\{(2, -1), (3, 2)\}$ in both domain and codomain. [5]

Question 4

The linear transformation $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 2 \\ 0 & -3 & 4 \\ 0 & -2 & 3 \end{pmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$ with respect to the standard basis.

(b) Find a basis for $\mathbb{R}^3$ consisting of eigenvectors of $\mathbf{A}$. [5]