

PART II

- (i) You should attempt no more than **THREE** questions from this part.
(ii) Each question carries 10 marks. The mark allocation for each section of a question is given in square brackets beside the section.
(iii) Start each question on a new page of your answer book.

Question 15

The set of complex numbers

$$S = \left\{ 1, -1, i, -i, \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}} \right\}$$

forms a group under multiplication. (You are NOT asked to prove this statement.)

- (a) Show that S is cyclic. [5]
(b) Find cyclic subgroups of S of orders 1, 2, 4 and 8. State whether there is a non-cyclic subgroup of S of order 4, giving a brief reason for your answer. [3]
(c) Give brief reasons why S is not isomorphic to either $S(\square)$ or \mathbb{Z}_8 . [2]

Question 16

This question concerns the set

$$S = \{(a, b, a-b, b-2a) : a, b \in \mathbb{R}\}.$$

- (a) Prove that S is a subspace of \mathbb{R}^4 . [3]
(b) Show that $\{(1, 1, 0, -1), (2, 1, 1, -3)\}$ is a basis for S , and state the dimension of S . [3]
(c) Find an orthogonal basis for S that includes the vector $(1, 1, 0, -1)$. [2]
(d) Express the vector $(2, 1, 1, -3)$ in terms of your orthogonal basis from part (c). [2]

Question 17

Determine whether each of the following series is convergent, naming any result or test that you use.

- (a) $\sum_{n=1}^{\infty} \frac{n^2 + 3}{2n^4 - n}$ [3]
(b) $\sum_{n=1}^{\infty} \frac{\sin n}{3n^3 + 1}$ [4]
(c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n - 1}$ [3]