

## PART I

- (i) You should attempt as many questions as you can in this part.
- (ii) Write your answers in the answer book provided, beginning each question on a new page.
- (iii) Questions in this part do not necessarily carry equal marks. The mark allocation is indicated for each question.

### Question 1

Draw a sketch of the graph of the function  $f$  defined by

$$f(x) = \frac{3x+1}{2-x}.$$

Your sketch should include:

- (a) any asymptotes for the graph.
- (b) any points where the graph crosses the axes.

[4]

### Question 2

The position vectors of the points  $A$ ,  $B$  and  $C$  are

$$\mathbf{a} = (4, -3), \mathbf{b} = (1, 3) \text{ and } \mathbf{c} = (2, 1).$$

- (a) Show that  $OC$  is perpendicular to  $AB$ .
- (b) Write down the position vector of a general point on the line  $AB$  and hence show that  $C$  lies on  $AB$ .
- (c) State the ratio in which  $C$  divides the line segment  $AB$ .

[5]

### Question 3

- (a) Determine the row-reduced form of the matrix

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 4 & -1 \end{pmatrix}.$$

- (b) Hence or otherwise find the solution set of the equations

$$x + 2y - z = 0,$$

$$2x - y + 3z = 0,$$

$$3x + 4y - z = 0.$$

[5]

### Question 4

The matrix  $\mathbf{A} = \begin{pmatrix} 5 & -3 \\ 4 & -3 \end{pmatrix}$  represents a linear transformation  $f$  with respect to the standard basis in both domain and codomain.

- (a) Determine the eigenvalues of  $f$ .
- (b) Find a basis for each eigenspace of  $f$ .

[4]

### Question 5

Determine the greatest lower bound of the set  $E$ , where

$$E = \left\{ 3 + \frac{2}{n^2} \mid n = 1, 2, 3, \dots \right\}$$

[4]