Question 18

The set of matrices

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & a+b \end{pmatrix} : a, b \in \mathbb{R}, \quad a(a+b) \neq 0 \right\}$$

forms a group under matrix multiplication.

We define an action $\wedge$ of G on $\mathbb{R}^2$ by

$$\begin{pmatrix} a & b \\ 0 & a+b \end{pmatrix} \wedge (x, y) = (ax, (a+b)y),$$

where $\begin{pmatrix} a & b \\ 0 & a+b \end{pmatrix} \in G$ and $(x, y) \in \mathbb{R}^2$.

(You are NOT asked to prove either of the statements above.)

(a) Find the orbits of

(i) $(1, 0)$,

(ii) $(0, 1)$,

(iii) $(1, 1)$,

and hence determine the set of orbits of the action. [5]

(b) Find the stabilizers of

(i) $(0, 1)$,

(ii) $(1, 1)$. [3]

(c) Find $\text{Fix}\left(\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}\right)$. [2]