

The following three questions are alternative Geometry questions for RESIT STUDENTS ONLY.

Question 11 R

Sketch, on three separate diagrams, each of the following Circles together with its image under inversion in the unit circle. If the image is a circle state its centre and radius.

- (a) S_1 : the line $y = -1$.
- (b) S_2 : the line $x = 2$.
- (c) S_3 : the circle with centre $(0, 4)$ and radius 3.

Mark each Circle carefully to distinguish it from its image.

[6]

Question 12 R

- (a) Find the affine transformation $t: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 2 \\ -1 \end{pmatrix},$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 4 \\ 2 \end{pmatrix},$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 5 \\ 3 \end{pmatrix}.$$

- (b) Find the inverse of t , giving your answer in the form

$$t^{-1}: x \mapsto Ax + b.$$

[5]

Question 19 R

In this question, C is the conic with equation

$$3x^2 + 2\sqrt{3}xy + y^2 - x + 3\sqrt{3}y + 3 = 0.$$

To help you to reduce the equation of C to standard form, you are given the following information. The eigenvalues of the matrix

$$A = \begin{pmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

are 0 and 4 and corresponding eigenvectors are $(1, -\sqrt{3})$ and $(\sqrt{3}, 1)$.

- (a) Write the equation of C in matrix form.

[1]

- (b) Write down a matrix P such that $P^T A P = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$.

[2]

- (c) Find the equation of the conic after the transformation $x \mapsto P^T x$, (i.e. $x = P x'$). DO NOT leave your answer in matrix form.

[3]

- (d) Find and specify a translation that reduces the equation of part (c) to standard form. Hence classify the conic.

[4]

[END OF QUESTION PAPER]