Question 9

The set

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} : a, b \in \mathbb{R}, \quad a \neq 0 \right\}$$

forms a group under matrix multiplication. (You are NOT asked to prove this result.) This question concerns the function ϕ defined by

$$\phi : G \to (\mathbb{R}, +)$$
$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mapsto \frac{b}{a}.$$

(a) Prove that ϕ is a homomorphism.

(b) Find the kernel of ϕ. [5]

Question 10

Find a matrix, **A**, associated with the projective transformation which maps the Points $[1, 0, 0]$, $[0, 1, 0]$, $[0, 0, 1]$ and $[1, 1, 1]$ to the Points $[1, 3, 2]$, $[2, 0, 1]$, $[0, 3, 0]$ and $[5, 0, 4]$, respectively. [5]

Question 11

(*RESIT STUDENTS* should attempt Question 11 R on page 9 of this paper.)

(a) Find the Möbius transformation that maps the points $3i$, $-2 - i$ and -4 to 0, 1 and ∞ respectively.

(b) Using the Möbius transformation that you found in part (a), show that $3i$, $-2 - i$, -4 and 0 lie on a generalised circle. [6]

Question 12

(*RESIT STUDENTS* should attempt Question 12 R on page 9 of this paper.)

(a) Find the equation of the projective conic that passes through the Points $[1, 0, 0]$, $[0, 1, 0]$, $[0, 0, 1]$, $[1, -1, 1]$ and $[1, 3, -2]$.

(b) Find the equation of the tangent at the Point $[1, 3, -2]$ to the projective conic in part (a). [5]

Question 13

The function f is defined by

$$f(x) = \begin{cases} 2 + 3x, & x < 1, \\ -3x^2 + 9x - 1, & x \geq 1. \end{cases}$$

Determine whether or not f is differentiable at the point 1. If it is differentiable, evaluate the derivative $f'(1)$. [5]

Question 14

Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3}{n^{\frac{1}{2}} 2^n} (x - 2)^n$$ [6]