Question 5

Determine the least upper bound of the set E, where

$$E = \left\{ 2 - \frac{1}{n^2} : n = 1, 2, 3, \ldots \right\}. \qquad [4]$$

Question 6

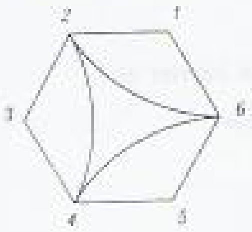
Determine whether each of the following series is convergent. (You should name any result or test that you use.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 3^n}{n!}$ $\geq \sum \frac{3^n}{(n-2)!} = n^{-n} \frac{(n+1)^2 3^{n+1}}{(n+1)!} \times \frac{\cancel{n!}}{n^2 \cancel{3^n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{2n + n^3}$ $= \frac{3(n+1)}{\cancel{n!}\, n^2} \not\to 0$ [5]

Question 7

The plane figure below comprises a regular hexagon and some circular arcs of equal radius.



(a) Write down the number of symmetries of the figure.

(b) List the symmetries of the figure in cycle notation, using the labelling of the vertex positions shown above.

(c) Write down a subgroup of the symmetry group of the figure that has order three. [5]

Question 8

The sets G and H are defined as follows:

$$G = \{1, 3, 7, 9, 11, 13, 17, 19\}; \; H = \{1, 11\}.$$

The set G forms a group with the operation multiplication modulo 20. (You are NOT asked to prove this statement.)

(a) Show that H is a subgroup of G.

(b) Find the cosets of H in G.

(c) Justify the statement that the quotient group G/H exists.

(d) Identify the quotient group G/H as isomorphic to $\mathbb{Z}_2$, $\mathbb{Z}_4$, K_4, or $\mathbb{Z}_8$. [6]

Question 9

Find a projective transformation which maps the Points $[1, 0, 0]$, $[0, 1, 0]$, $[0, 0, 1]$ and $[1, 1, 1]$ to the Points $[1, 2, 0]$, $[2, 0, 3]$, $[1, 1, 1]$ and $[-2, 1, 3]$, respectively. [4]