ANALYSIS B

Question 21

(a) The function f is defined on $[0,1]$ by

$$f(x) = \begin{cases} 1, & x = 0, \\ 2x, & 0 < x < 1, \\ 0, & x = 1. \end{cases}$$

(i) Sketch the graph of f.

(ii) Determine the values of the Riemann sums $L(f,P)$ and $U(f,P)$ for the partition P of $[0,1]$ where

$$P = \left\{ \left[0, \tfrac{1}{3}\right], \left[\tfrac{1}{3}, \tfrac{1}{2}\right], \left[\tfrac{1}{2}, 1\right] \right\}.$$ [5]

(b) Show that

$$\int \frac{dx}{x(\log_e x)^{3/4}} = 4(\log_e x)^{1/4},$$

and hence determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\log_e n)^{3/4}}$$

converges or diverges. [5]

Question 22

(a) Determine the Taylor polynomial $T_3(x)$ for the function $f(x) = (3+x)^{5/2}$ at 1. Show that $T_3(x)$ approximates $f(x)$ to within 10^{-2} on the interval $[1,2]$. [6]

(b) Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{4^n}{n}(x-1)^n.$$ [4]

[END OF QUESTION PAPER]