GEOMETRY

Question 19

In this question C is the conic with equation

$$11x^2 + 96xy + 39y^2 + 10x + 180y - 75 = 0.$$

To help you to reduce the equation of C to standard form, you are given the following information:

the eigenvalues of the matrix $\mathbf{A} = \begin{pmatrix} 11 & 48 \\ 48 & 39 \end{pmatrix}$ are 75 and -25 with corresponding eigenvectors $(3, 4)$ and $(-4, 3)$.

(a) Write the equation of C in matrix form. [1]

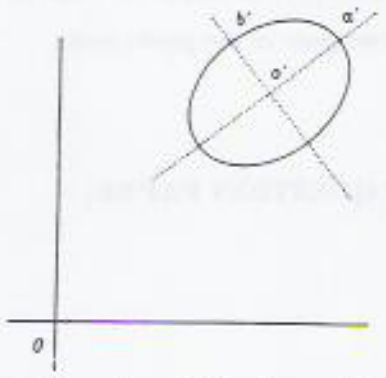
(b) Write down a matrix $\mathbf{P}$ such that $\mathbf{P}^T\mathbf{A}\mathbf{P} = \begin{pmatrix} 75 & 0 \\ 0 & -25 \end{pmatrix}$. [2]

(c) Find the equation of the conic after the transformation $\mathbf{x} \longmapsto \mathbf{P}^T\mathbf{x}$ (i.e. $\mathbf{x} = \mathbf{P}\mathbf{x}'$). Do NOT leave your answer in matrix form. [3]

(d) Find a translation that reduces the equation of part (c) to standard form. Hence classify the conic C. [4]

Question 20

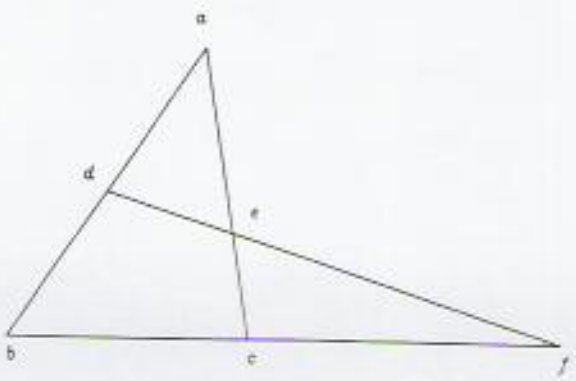
(a) The figure below shows an ellipse with centre at the point o', and with a' being an end of the major axis and b' an end of the minor axis. The points o', a' and b' have coordinates $(6, 10)$, $(10, 12)$ and $(5, 12)$ respectively.



(i) Find an affine transformation of the plane which maps the origin to o', the point $(1, 0)$ to a' and the point $(0, 1)$ to b'.

(ii) Use your answer to part (i) to find an affine transformation that maps the ellipse in the figure to the circle $x^2 + y^2 = 1$. [7]

(b) In the figure below, d is the mid-point of ab and the point e divides ac in the ratio $3 : 2$. Find the ratio $bc : cf$, quoting any theorems that you use.



[3]