

Question 10

- (a) Find an affine transformation, t , of the plane which sends the triangle with vertices $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to the triangle with vertices $\begin{pmatrix} -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, preserving the given order of the vertices.
- (b) Find the inverse of t , giving your answer in the form

$$t^{-1}: \mathbf{x} \mapsto \mathbf{Ax} + \mathbf{b}. \quad [5]$$

Question 11

Prove that

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 \cos x}$$

exists and determine its value.

[5]

Question 12

The function f is defined on $[0, 1]$ by

$$f(x) = \begin{cases} 1 - 2x, & 0 \leq x < \frac{1}{2}, \\ 1 + x, & \frac{1}{2} \leq x < 1, \\ 1, & x = 1. \end{cases}$$

- (a) Sketch the graph of f .
- (b) Determine the values of the Riemann sums $L(f, P)$ and $U(f, P)$ for the partition P of $[0, 1]$ where

$$P = \left\{ \left[0, \frac{1}{2}\right], \left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{2}{3}, 1\right] \right\}. \quad [5]$$