Question 16

(a) Is the following function continuous at 1?

$$f(x) = \begin{cases} e^x, & x \leq 1, \\ x+1, & x > 1. \end{cases}$$

Justify your answer. [2]

(b) Determine all the points at which the following function is continuous.

$$f(x) = \begin{cases} 1 + 2x + x^4, & x \leq 0, \\ \cos^2 x + \sin x, & x > 0. \end{cases}$$

[5]

(c) Show that there is a value of x, $x \in [0, \pi]$, for which $x \cos^2 x = 1$. State clearly any results that you use. [3]

GROUP THEORY

Question 17

Let $G = \left\{ \begin{pmatrix} a & 0 \\ b & a \end{pmatrix} : a, b \in \mathbf{R},\ a \neq 0 \right\}$.

(a) Show that G is a group under matrix multiplication. (You may assume that multiplication of matrices is associative.) [4]

(b) Show that the function $\phi : G \longrightarrow (\mathbf{R}, +)$ given by $\begin{pmatrix} a & 0 \\ b & a \end{pmatrix} \longmapsto b/a$ is a homomorphism, and find the kernel of ϕ. [3]

(c) Explain why $H = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} : a \in \mathbf{R}, a \neq 0 \right\}$ is a normal subgroup of G and why $G/H \cong (\mathbf{R}, +)$. [3]