PART II

(i) *You should attempt* **not more than FOUR** *questions from this part.*

(ii) *Each question carries 10 marks. The mark allocation for sections of questions is given in square brackets beside each section.*

(iii) *Start each question on a new page of your answer book.*

(iv) *The questions are grouped together according to subject area.*

LINEAR ALGEBRA

Question 13

The matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ -2 & 2 & 3 \end{pmatrix}$ represents the linear transformation $f : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ with respect to the standard basis in both the domain and the codomain.

(a) Determine the characteristic equation of $\mathbf{A}$ and solve this equation to find the eigenvalues of f. [3]

(b) Find bases for the eigenspaces of f. [4]

(c) The matrix, $\mathbf{C}$, of f with respect to the eigenvector basis in domain and codomain can be expressed as $\mathbf{C} = \mathbf{P}^T\mathbf{A}\mathbf{P}$. Find the matrix $\mathbf{P}$ and write down the matrix $\mathbf{C}$. [3]

Question 14

In this question, V denotes the vector space of all polynomials in x of degree at most 2, that is

$$V = \{p(x) : p(x) = ax^2 + bx + c, a, b, c \in \mathbf{R}\}.$$

(You are NOT asked to prove that V is a vector space.)

(a) Determine whether each of the following subsets of V is a subspace.

(i) $S = \{p(x) : p(x) = ax^2 - ax, a \in \mathbf{R}\}$

(ii) $T = \{p(x) : p(x) = ax^2 + bx + c, a, b, c \in \mathbf{R},\ a + b + c = 2\}$ [4]

(b) Show that the function

$$t : V \longrightarrow V$$
$$p(x) \longmapsto x\,p(0) - p(1)$$

is a linear transformation, and determine the kernel of t in the form $\mathrm{Ker}(t) = \{p(x) : p(x) = ax^2 + bx + c \in V$, some conditions on a, b, $c\}$. [6]

ANALYSIS A

Question 15

Determine whether each of the following series is convergent. (You should name any result or test that you use.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n n!}$ [3]

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2 + n^2}$ [2]

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(2\pi/n)}{4n^2 + 1}$ [5]