Question 20

Let XYZ be the triangle of reference in $\mathbb{RP}^2$ and let $P = [\alpha, \beta, \gamma]$ be a general Point not lying on any side of the triangle. The Lines XP and YZ intersect in the Point A, YP and XZ intersect in B and ZP and XY intersect in C.

(a) Find the equation of the Line XP and the co-ordinates of the Point A. Hence, or otherwise, write down the equations of the Lines YP and ZP and the coordinates of the Points B and C. [4]

(b) Find the Points of intersection :

F of BC and YZ,
G of AC and XZ,
H of AB and XY. [3]

(c) Find the equation of the Line FGH. (You may assume that F, G, H are collinear.) [2]

(d) If in a particular case, the equation of the Line FGH is $x + y + z = 0$, what is the Point P? [1]

ANALYSIS B

Question 21

Prove that the following functions are differentiable everywhere and determine the derived function of each.

(a) $f(x) = 2x^5 + 4x^3 - 3x^2$ [2]

(b) $g(x) = \begin{cases} x^4\cos(2/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$ [5]

(c) $h(x) = \begin{cases} 2x^5 + 4x^3 - 3x^2, & x \leq 0, \\ x^4\cos(2/x), & x > 0. \end{cases}$ [3]

Question 22

This question is about the linear flow for which the velocity function is

$$V(x, y) = (9x - 4y, \quad 24x - 11y).$$

(a) Write down

(i) the matrix $\mathbf{A}$ of the flow;

(ii) the first order differential equations satisfied by the co-ordinate functions f and g of any flow function $\alpha = (f, g)$ of this flow;

(iii) a second order differential equation satisfied by both f and g. [3]

(b) Find the general solution of the differential equation in part (a) (iii). [2]

(c) Determine the flow function α for V, and find the particular solution that satisfies $\alpha(0) = (1, 0)$. [5]

[END OF QUESTION PAPER]