PART I

(i) *You should attempt as many questions as you can in this part.*

(ii) *Write your answers in the answer book provided, beginning each question on a new page.*

(iii) *Questions in this part do not necessarily carry equal marks. The mark allocation is indicated in square brackets beside each question.*

Question 1

Draw a sketch of the graph of the function $f$ defined by

$$f(x) = \frac{3x-4}{x-2}.$$

Your sketch should include:

(a) any asymptotes to the graph;

(b) points where the graph crosses the axes. [4]

Question 2

The position vectors of points $a$ and $b$ are $\mathbf{a} = (-2, 1)$ and $\mathbf{b} = (1, -3)$ respectively.

(a) Draw a sketch showing the points $a$ and $b$ in the plane, and the line $l$ through $a$ and $b$.

(b) The point $c$ has position vector

$$\mathbf{c} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}.$$

(i) Does $c$ lie on $l$? Justify your answer briefly.

(ii) Find the length of $Oc$.

(iii) Show that $Oc$ is perpendicular to $l$. [5]

Question 3

Let $S = \{(0,1,2),(1,1,1),(3,1,-1)\}$ be a set of three vectors in $\mathbf{R}^3$, and let

$$T = \{(x,y,z) \in \mathbf{R}^3 : \ x - 2y + z = 0\}.$$

You may assume that $T$ is a subspace of $\mathbf{R}^3$.

(a) Show that $S$ is a linearly dependent set, but that $\{(0,1,2),(1,1,1)\}$ is a linearly independent set.

(b) Prove that $\{(0,1,2),(1,1,1)\}$ is a basis for $T$.

(c) Hence or otherwise show that $\langle S \rangle = T$. [7]

Question 4

(a) Determine the row-reduced form of the matrix

$$\begin{pmatrix} 1 & 3 & 7 \\ 3 & 2 & 7 \\ -1 & 1 & 1 \end{pmatrix}.$$

(b) Hence, or otherwise, find the solution set of the equations

$$\begin{aligned} x + 3y + 7z &= 0, \\ 3x + 2y + 7z &= 0, \\ -x + y + z &= 0. \end{aligned}$$

[5]