

Question 13

- (i) Show that the following formula takes truth value 1 under all interpretations of its symbols.

$$(((x = y \vee \exists x x = y) \rightarrow \forall x(x = y \vee \exists x x = y)) \rightarrow (\neg \forall x(x = y \vee \exists x x = y) \rightarrow \neg x = y)) \quad [3]$$

- (ii) The following is a correct (but contorted) proof from which the assumption numbers have been omitted.

(1)	$(\psi \rightarrow \theta)$	Ass	1
(2)	$\forall x(\theta \& \phi)$	Ass	2
(3)	$(\theta \& \phi)$	UE, (2)	2
(4)	$\exists x(\psi \rightarrow \theta)$	Ass	4
(5)	$(\psi \rightarrow \theta)$	Taut, (1), (3)	1, 2
(6)	$\exists x(\psi \rightarrow \theta)$	EI, (5)	1, 2
(7)	$\exists x(\psi \rightarrow \theta)$	EH, (6)	2, 4
(8)	$((\psi \rightarrow \theta) \rightarrow \exists x(\psi \rightarrow \theta))$	CP, (6)	2
(9)	$((\psi \rightarrow \theta) \rightarrow \exists x(\psi \rightarrow \theta))$	Taut, (7)	2, 4

- (a) Write down the assumptions in force on each line. [2½]

- (b) Write down the tautology used on line (5). [½]

- (c) For each of the following lines, write down whether the proof would still be correct were the line to be added to it.

(A)	2, 4 (10)	$(\psi \rightarrow \theta)$	EH, (5)	NO
(B)	1 (10)	$\exists x(\psi \rightarrow \theta)$	EI, (1)	YES

Answer YES or NO. [2]

- (iii) The rule SR states that:

Suppose that each of the terms τ_1 and τ_2 may be freely substituted for the variable v in the formula ϕ . Let ϕ_1 be a formula which results when some (but not necessarily all) of the free occurrences of v in ϕ are replaced by τ_1 , and let ϕ_2 be the formula that results when these same occurrences are replaced by τ_2 . Then if on two lines of a formal proof the formulas $\tau_1 = \tau_2$ and ϕ_1 occur, on any subsequent line we may derive ϕ_2 , and ϕ_2 will depend on all the assumptions in force on the lines on which the formulas $\tau_1 = \tau_2$ and ϕ_1 occur.

Give a suitable example from everyday mathematics to show that the rule is no longer valid if the underlined passage, which gives a condition on the terms τ_1 and τ_2 , is omitted. [3]

A