Question 8

(i) Determine the continued fraction of $\sqrt{12}$ and hence find two positive solutions of the Diophantine equation

$$x^2 - 12y^2 = 1.$$ [6]

(ii) For each of the numbers 560, 570 and 580 decide whether or not it can be expressed as a sum of two squares. For any which you claim cannot be so expressed, justify your answer. For any which can be so expressed, find two different expressions as a sum of two positive squares. [5]

The following two questions are for **RESIT STUDENTS ONLY**

Question 4R

(i) Prove Fermat's Little Theorem (FLT): If p is a prime and a is any integer with $\gcd(a, p) = 1$, then $a^{p-1} \equiv 1 \pmod{p}$. [6]

(ii) Use Wilson's Theorem to:

(a) find the smallest prime divisor of $28! + 1$;

(b) prove that $n! + 1$ is composite for infinitely many values of n. [5]

Question 7R

(i) Determine the continued fraction of $2 + \sqrt{12}$. Write down the convergents C_0, C_1, C_2, C_3 and C_4 and estimate the accuracy $|C_3 - (2 + \sqrt{12})|$ of C_3. [6]

(ii) Determine the irrational number which has periodic continued fraction $[3; 1, \overline{1, 2}]$. [5]