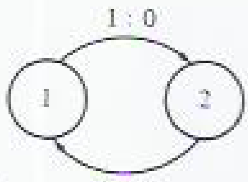
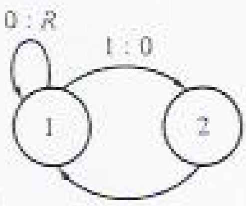
Question 9

(i) We wish to design a Turing machine which, using monadic notation, takes as input a pair (m, n) of positive integers in standard starting position (on an otherwise blank tape) and which halts on a blank tape.

(a) Explain why each of the Turing machines below is *not* suitable for this task. (Your answer may include sequences of configurations for appropriate test data.)



(1)



(2) [4]

(b) Give the flowgraph of a machine which correctly performs the task. [2]

(ii) Give the complete flowchart of an Abacus machine program which has the effect shown in the following block diagram. (You may use extra registers, assumed empty initially, if you wish.)

$$\downarrow$$
$$\boxed{\begin{array}{rl} |[m]-[n]| & \longrightarrow p \\ [m]+[n] & \longrightarrow m \\ 0 & \longrightarrow n \end{array}}$$
$$\downarrow$$

where $[p] = 0$ initially.

where $|x - y| = \begin{cases} x - y, & \text{if } x \geq y, \\ y - x, & \text{if } x < y. \end{cases}$

[5]

Question 10

(i) Let h be the function

$$\mathrm{Pr}[\mathrm{Cn}[s, s], \mathrm{Cn}[\exp, \mathrm{Cn}[s, \mathrm{id}_2^3], \mathrm{id}_3^3]]$$

where exp is the function defined by $\exp(x, y) = x^y$.

Compute the values of

(a) $h(3, 0)$,

(b) $h(3, 2)$. [4]

(ii) In this part you may present your arguments using either formal or informal definitions.

(a) Show that the function exp, as in part (i), is primitive recursive by defining it in terms of the sum function and the initial functions. [2]

(b) Show that the function f of 3 arguments defined by

$$f(x_1, x_2, x_3) = x_1^{(x_2^{x_3})}$$

is primitive recursive by defining it in terms of exp, the initial functions and, if you wish, the sum and product functions. [3]

(iii) Write down the pairs (x_1, x_2) of natural numbers for which $\mathrm{Mn}[f](x_1, x_2)$ is defined, where f is the function defined by

$$f(x_1, x_2, y) = (x_2 \dot{-} (x_1 + y)) + y \cdot (4 \dot{-} x_2).$$ [2]