Question 13

(i) Show that the following formula takes truth value 1 under all interpretations of its symbols.

$$(((x = y \vee \forall x\, x = y) \longrightarrow \exists x(x = y \vee \forall x\, x = y)) \longrightarrow (-\exists x(x = y \vee \forall x\, x = y) \longrightarrow -x = y))$$ [3]

(ii) The following is a correct (but contorted) proof from which the assumption numbers have been omitted.

(1)	$(\psi \longrightarrow -\phi)$	Ass	1
(2)	$\forall x(-\theta \vee \psi)$	Ass	2
(3)	$(-\theta \vee \psi)$	UE, (2)	2
(4)	$\exists x(\psi \longrightarrow -\phi)$	Ass	4
(5)	$(\phi \longrightarrow -\psi)$	Taut, (1)	1
(6)	$(\phi \longrightarrow -\theta)$	Taut, (3),(5)	1, 2
(7)	$\exists x(\phi \longrightarrow -\theta)$	EI, (6)	1, 2
(8)	$(\forall x(-\theta \vee \psi) \longrightarrow \exists x(\phi \longrightarrow -\theta))$	CP, (7)	1
(9)	$(\forall x(-\theta \vee \psi) \longrightarrow \exists x(\phi \longrightarrow -\theta))$	EH, (8)	4

(a) Write down the assumptions in force on each line. [$2\frac{1}{2}$]

(b) Write down the tautology used on line (6). [$\frac{1}{2}$]

(c) For each of the following possible line (10)s, write down whether the proof would still be correct were the line to be added.

(A) 1(10) $\exists x(\psi \longrightarrow -\phi)$ EI, (1) Yes

(B) 1(10) $\forall x(\phi \longrightarrow -\psi)$ UI, (5) NO.

Answer YES or NO. [2]

(iii) Explain why the following proof of the theorem $\forall x\, x = 0 \vdash \forall y\, y = 0$ is incorrect and give a correct formal proof of the theorem.

1(1)	$\forall x\, x = 0$	Ass
1(2)	$x = 0$	UE, (1)
1(3)	$\forall y\, y = 0$	UI, (2)

[3]

Question 14

(i) Which of the following terms are freely substitutable for x in the formula

$$\forall y(\forall x \exists t(x + y) = t \vee \exists z(z + t) = x)?$$

(a) $(z + x)$ ✗

(b) $(t + x)$ ✓

(c) $(0 + y)$ ✗

Answer YES or NO in each case. [2]

(ii) Give a formal proof to show that

$$\forall x(\psi \longleftrightarrow -\theta) \vdash (\theta \longrightarrow (\exists x(-\psi \longrightarrow \phi) \longrightarrow -\forall x - (\phi \,\&\, \theta)))$$

where the variable x does not occur free in θ. Indicate the step(s) of your proof which require the condition on θ. [9]