

Question 6

- (i) Decide whether or not the quadratic congruence

$$2x^2 + 7x + 4 \equiv 0 \pmod{23}$$

has solutions.

[4]

- (ii) Evaluate the Legendre symbol $(166/67)$.

[3]

- (iii) Use Gauss's Lemma to show that 2 is a quadratic non-residue of any prime p for which $p \equiv 5 \pmod{8}$.

[4]

Question 7

- (i) Determine the continued fraction of $\frac{50}{37}$ and hence find the general solution of the linear Diophantine equation

$$50x - 37y = 2.$$

[5]

- (ii) Determine the irrational number which has the periodic continued fraction $[5; 1, \overline{2, 3}]$.

[6]

Question 8

- (i) Determine the continued fraction of $\sqrt{11}$ and, from its convergents, determine three solutions of the Diophantine equation

$$x^2 - 11y^2 = 1.$$

[6]

- (ii) Use the method of infinite descent to show that the Diophantine equation

$$x^3 + 2y^3 = 4z^3$$

has no positive solutions.

[5]