Question 11

(i) A Turing machine has a configuration with left number 37 and right number 61. Find the left and right numbers of the configuration which results when the scanning head is moved one square to the right, and draw the resulting configuration. [3]

(ii) In parts (a), (b) and (c) below, subject to the provisos in parts (a) and (b), you may use any of the recursive functions, or results about them, given in the Logic Handbook, without proving that they are recursive.

(a) Show that the condition "x is even" on natural numbers x is primitive recursive. You may *not* use the functions e or d in the Logic Handbook. [$2\frac{1}{2}$]

(b) Show that if g_1, g_2 and g_3 are primitive recursive functions and C_1 and C_2 are mutually exclusive primitive recursive conditions on pairs (x, y) of natural numbers, then the function f defined by

$$f(x,y) = \begin{cases} g_1(x,y), & \text{if } C_1, \\ g_2(x,y), & \text{if } C_2, \\ g_3(x,y), & \text{otherwise,} \end{cases}$$

is also primitive recursive. You may *not* use either of the results on page 3 of the Logic Handbook of which this is a special case. [$2\frac{1}{2}$]

(c) Show that the function f defined by

$$f(x,y) = \begin{cases} x^3, & \text{if } x \cdot y \text{ is odd}, \\ x + 4y, & \text{if } 2x = 3y, \\ 7, & \text{otherwise,} \end{cases}$$

is primitive recursive. [3]

Question 12

In this question you may use any of the recursive functions, or results about them, given in the Logic Handbook without proving that they are recursive. You may also give your answers as informal definitions.

(i) Show that the function c defined by

$$c(x,y) = \begin{cases} 1, & \text{if } y^3 \le x, \\ 0, & \text{otherwise,} \end{cases}$$

is primitive recursive. [3]

(ii) Show that if f is a primitive recursive function of 2 arguments, then the function g defined by

$$g(x, v, w) = \sum_{z=v}^{w} f(x, z)$$

is also primitive recursive. [3]

(iii) By summing the values of $c(x, z)$ for appropriate values of z, or otherwise, show that the function k defined by

$$k(x) = \left[x^{1/3}\right],$$

i.e. the greatest integer less than or equal to the cube root of x (so that e.g. $k(0) = 0$, $k(1) = k(2) = \ldots = k(7) = 1$, $k(8) = 2$) is primitive recursive. [5]