Question 15

For each of the following sentences, decide whether or not it is a theorem of Q. If it is a theorem of Q, write down a formal proof showing this. If it is not a theorem of Q, justify this. *(You may use without proof the fact that all the axioms of Q are true under the interpretations N^* and N^{**} given in the Logic Handbook.)*

(i) $\forall x\, \forall y((x \cdot y) + 0) = ((x + 0) \cdot (y + 0))$

(ii) $\exists y\, \forall x(y + x) = x$

(iii) $\exists x\, \forall y(y + x) = y$ [11]

Question 16

(i)   Give brief explanations of *each* of the following:

    (a)   Church's Thesis;

    (b)   a *decidable* theory;

    (c)   the theory *arithmetic*.   [6]

(ii)   Which theorem (or theorems) of the course give(s) an answer to Leibniz's Question:

Is there an algorithm for deciding which statements of number theory are true?
Explain why the theorem(s) answer(s) the question.   [5]

(Your answer may include references to any of the theorems listed in the Logic Handbook.)