				STRATEGY
1	(1)	$\forall x\,(\phi \leftrightarrow -\psi)$	Ass	Write down ass^ns
1	(2)	$(\phi \leftrightarrow -\psi)$	UE(1)	remove $\forall x$
3	(3)	$\exists x\,(\psi \vee \theta)$	Ass	Want $(A \to B)$, intro A
4	(4)	$(\psi \vee \theta)$	Ass	introd. χ if got $\exists x \chi$
5	(5)	$\forall x\,(\phi \,\&\, -\theta)$	Ass	Want $-A$, intro. A
5	(6)	$(\phi \,\&\, -\theta)$	UE (5)	remove $\forall x$
4,5	(7)	ψ	Taut (4)(6)	Want $-A$, get
1, 5	(8)	$-\psi$	Taut $(\overset{2}{\ })(\overset{6}{\ })$	$(\chi \,\&\, -\chi)$ by Taut
1,4,5	(9)	$\psi \,\&\, -\psi$	Taut (7)(8)	
1,4	(10)	$(\forall x\,(\phi \,\&\, -\theta) \to (\psi \,\&\, -\psi))$	CP (9)	CP + Taut allow $-A$
1,4	(11)	$-\forall x\,(\phi \,\&\, -\theta)$	Taut (10)	
1,3	(12)	$-\forall x\,(\phi \,\&\, -\theta)$	EH (11)	use EH to change ass^n
1	(13)	$(\exists x\,(\psi \vee \theta) \to -\forall x\,(\phi \,\&\, -\theta)$	CP (12)	

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(15) We wish to know whether, each of the following is a theorem or not

(i) $\forall x \forall y\,((x \cdot y) + \underline{0}) = ((x + \underline{0}) \cdot y)$

(ii) $\forall y \forall z\,(\underline{0} + (y \cdot z)) = ((\underline{0} + y) \cdot z)$

Intuitively: (i) looks O.K — try for a proof. (ii) looks wrong — try for a counter-example.

(ii) is NOT a theorem as there is an interpretation N^{**} where it is false.

$$\underline{0} + (\underline{0} \cdot \beta) = \underline{0} + \beta = \alpha$$
$$(\underline{0} + \underline{0}) \cdot \beta = \underline{0} \cdot \beta = \beta$$

(i) is a theorem: the informal argument is

$\forall x\ (x + \underline{0}) = x$ AND $(x + \underline{0}) = x$

$(x + 0) = x$ $(x \cdot y) = (x \cdot y)$

$((x \cdot y) + 0) = (x \cdot y)$ $((x + 0) \cdot y) = (x \cdot y)$

$$((x \cdot y) + 0) = ((x + 0) \cdot y)$$

$\forall y$ ~~~~~~~~

$\forall x \forall y$ ~~~~~~~~