

$prod(x,y) = x \cdot y$
 \uparrow
 (ii) $h = Pr[C_n[s,z], C_n[prod, C_n[s, id_1^3], id_3^3]]$
 Compute $h(3,0), h(3,2)$.

Recall $h = Pr[f, g]$ means $h(x,0) = f(x)$
 $h(x, s(y)) = g(x, y, h(x, y))$

$$\begin{aligned}
 \text{thus } h(3,0) &= f(3) = 1 \\
 h(3,2) &= h(3, s(1)) = g(3, 1, h(3,1)) \\
 &\quad \uparrow \\
 &\quad y=1 \\
 h(3,1) &= h(3, s(0)) = g(3, 0, h(3,0)) \\
 &\quad \quad \quad \uparrow \\
 &\quad \quad \quad 1 \cdot 1 = 1 \\
 h(3,2) &= g(3, 1, 1) \\
 &\quad \quad \quad \uparrow \\
 &\quad \quad \quad 2 \cdot 1 = 2.
 \end{aligned}$$

(i) (a) Show $prod$ is prim rec. by defining in terms of sum and initial fns.

$$\begin{aligned}
 prod(x,0) &= 0 = z(x) \\
 prod(x, s(y)) &= x \cdot (y+1) = xy + x \\
 &\quad \quad \quad \leftarrow \text{informal.}
 \end{aligned}$$

$$\begin{aligned}
 prod &= Pr[z, sum(h(x,y), x)] \\
 &= Pr[z, C_n[sum, id_1^3, id_3^3]] \text{ formal}
 \end{aligned}$$

(b) Show $f(x,y) = (x+1)^y$ is prim rec. in terms of prod and initial fns.

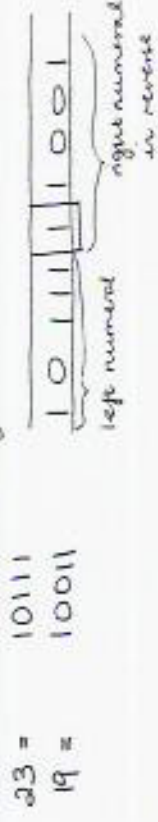
$$\begin{aligned}
 f(x,0) &= (x+1)^0 = 1 = C_n[s,z] \\
 f(x,y+1) &= (x+1)^{y+1} = (x+1)^y \cdot (x+1) = f(x,y) \cdot (x+1) \\
 g(x,y,f(x,y)) &= C_n[prod, C_n[s, id_1^3], id_3^3]
 \end{aligned}$$

(ii) Write down the pairs (x_1, x_2) for which $Mn[F](x_1, x_2)$ is defined, for $f(x_1, x_2, y) = (x_1 + y) \cdot (x_2 + y)$
 $Mn[F] = \{ \text{the smallest } y \text{ for which } f(x_1, x_2, y) \text{ is defined otherwise} \}$

$$\begin{aligned}
 \text{Let } f(x_1, x_2, y) &= 0; \text{ when } (x_1 + y) \cdot (x_2 + y) = 0 \\
 &\quad \quad \quad \therefore x_1 + y = 0 \text{ or } x_2 + y = 0
 \end{aligned}$$

But $x_1 + y = 0 \Rightarrow x_1 = 0, y = 0$ Thus $Mn[F]$ defined for (x_1, x_2) with $x_1 = 0$ or $x_2 = 0$ or both.

(ii) Draw the configuration of a Turing machine tape with left number 23, right number 19



(ii) We may use all results/fns in the Handbook

(a) Show ' $x=y$ ' is prim recursive.

$$\begin{aligned}
 \text{Let } f(x,y) &= sg(x-y) \\
 \text{then if } y > x & \quad x+y = 0, \quad f(x,y) = sg(0) = 1 \\
 y = x & \quad x-y = 0 \\
 y < x & \quad x-y > 0 \quad \} \quad f(x,y) = 0
 \end{aligned}$$

\therefore the characteristic fn of the condⁿ is pr so the condⁿ is prim rec.

$$\text{(b) Show } f(x,y) = \begin{cases} x-y & \text{if } x \leq y \\ x+y & \text{if } x > y \end{cases} \text{ is p.r.}$$

$$f(x,y) = \begin{cases} g_1 & \text{if } C_1 \\ g_2 & \text{if } C_2 \\ g_3 & \text{otherwise} \end{cases}$$

g_1, g_2, g_3 clearly p.r. fns.
 C_1, C_2 otherwise p.r. condⁿs
 C_1, C_2, C_3 exhaustive

All that remains is to show:

C_1, C_2 mutually exclusive: if $x \leq y$ then $x < y+1 \leq 2y$
 C_2 p.r. $Eg(x, 2y+1)$ is the characteristic fn.
 more formally $C_n[Eq, id_1^2, C_n[sum, id_2^2, id_3^2], sum, s(z(id_1^2))]$
 this will do.

$$C_n[s, C_n[z, id_1^2]]$$