⑥ Determine whether or not the quad. congruence
$2x^2 - 5x - 2 \equiv 0 \pmod{19}$

has soln^s

Solns $\Leftrightarrow y^2 \equiv a \pmod{19}$ a = discriminant
$\Leftrightarrow y^2 \equiv 3 \pmod{19}$ $= 5^2 + 4.2.2 = 41$
ie is 3 a quad. res of 19 ?

Several methods Gauss: $S = \{3, 6, 9, \boxed{12}, \boxed{15}, \boxed{18}, 21^2, 24^5, 27^8\}$
which exceed $p - \frac{1}{2} = 9$? 3 of them
$(a/p) = -1^3 = -1$ $\left(\frac{p-1}{2}\right)a$
ie No solns.

Euler $a^{\frac{(p-1)}{2}} \equiv 3^9 \pmod{19}$
$\equiv 8^3 \pmod{19} \equiv 7.8 \equiv 56 \equiv -1 \pmod{19}$ No solns

Trial + error $1^2, 2^2, 3^2, 4^2, 5^2 = 6, 6^2 = 17, 7^2 = 11, 8^2 = 7, 9^2 = 5$ No solns
(Only need to check this far as solns. occur in pairs)

(ii) Evaluate: $\left(\frac{143}{103}\right) = -\left(\frac{103}{43}\right) = -\left(\frac{17}{43}\right) = -\left(\frac{43}{17}\right)$
$= -\left(\frac{9}{17}\right) = -\left(\frac{3^2}{17}\right) = -1$.

(iii) Determine all odd primes for which $p-2$ is a quad. res modulo p $\left(\frac{p-2}{p}\right) = \left(\frac{-2}{p}\right) = 1$ if $p \equiv 1$ or $3 \pmod 8$

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⑦ Determine the continued fraction of $65/19$; hence find general soln of $65x + 19y = 3$

$65 = 3.19 + 8$
$19 = 2.8 + 3$
$8 = 2.3 + 2$
$3 = 1.2 + 1$
$2 = 2.1$

$\therefore \frac{65}{19} = 3 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + 1/2}}} = [3; 2, 2, 1, 2]$

the convergents are $\frac{3}{1}, \frac{7}{2}, \frac{17}{5}, \frac{24}{7}, \frac{65}{19}$

thus $x = q_{k-1} = 7$ $y = -p_{k-1} = -24$
then $ax + by = (-1)^{k-1} = -1^3 = -1$
$65(7) + 19(-24) = -1$, $65(-7) + 19(24) = 1$

So we have a soln of $65x + 19y = 1$
" " " " " " " " $= 3$
ie $x_0 = -21$ $y_0 = 72$
General soln is $x = -21 + 19t$
$y = 72 - 65t$

This general soln is in handbook.

(ii) Determine the continued fraction of $\frac{\sqrt{12} + 9}{3}$ and write down $C_0 \cdots C_4$. Which is the first ${}^3C_1$ accurate to within 0.001

$x = \frac{\sqrt{12} + 9}{3}$ $\therefore [x] = 4 = a_0$

$x_1 = \frac{1}{x - [x]} = \frac{1}{\frac{\sqrt{12}+9}{3} - 4} = \frac{3}{\sqrt{12} - 3} \times \frac{\sqrt{12}+3}{\sqrt{12}+3} = \sqrt{12} + 3$ $a_1 = 6$

$x_2 = \frac{1}{x_1 - [x_1]} = \frac{1}{\sqrt{12} - 3} \times \frac{\sqrt{12}+3}{\sqrt{12}+3} = \frac{\sqrt{12}+3}{3}$ $a_2 = 2$

$\therefore x_3 = \frac{1}{x_2 - [x_2]} = \frac{3}{\sqrt{12} + 3 - 6} = \frac{3}{\sqrt{12} - 3} = x_1$

$\therefore x = [4; 6, \overline{2}]$

Check $x = [4; 6, \overline{2}] = 4 + \frac{1}{y}$ where $y = [6; \overline{2}]$

$y = 6 + \frac{1}{2 + \frac{1}{y}} = 6 + \frac{y}{2y+1} = \frac{13y + 6}{2y + 1}$ $\therefore 2y^2 - 12y - 6 = 0$
$y^2 - 6y - 3 = 0$
$y = \frac{6 + \sqrt{36 + 12}}{2}$

$\therefore x = 4 + \frac{1}{3 + \sqrt{12}} = \frac{13 + 4\sqrt{12}}{3 + \sqrt{12}} \times \frac{3 - \sqrt{12}}{3 - \sqrt{12}} = \frac{-9 - \sqrt{12}}{-3} = \frac{\sqrt{12} + 3}{3}$

The convergents are $\frac{4}{1}, \frac{25}{6}, \frac{54}{13}, \frac{349}{84}, \frac{752}{181}$

$\frac{1}{\cdot} < |x - \cdot| < \frac{1}{\cdot}$ So $\frac{1}{\cdot} = \frac{1}{181} > 0.001$ but $\frac{1}{\cdot} = \frac{1}{\cdot}$