(8)(i) Find 3 positive solutions of the Diophantine eqn.

(Pell's Eqn) $x^2 - 15y^2 = 1$ (Hint: $\sqrt{15} = [3; \overline{1, 6}]$)

the convergents of $\sqrt{15}$ are $\frac{3}{1}$, $\frac{4}{1}$, $\frac{27}{7}$, etc. $\frac{31}{8}$, $\frac{213}{55}$, $\frac{244}{63}$

the period of expansion is 2 so all pos. solns are

$x = p_{kn-1}$, $y = q_{kn-1}$ ie $x = p_1, p_3, p_5$

$y = q_1, q_3, q_5$

So $(4,1)$, $(31,8)$, $(244,63)$

(ii) For each of these numbers determine whether or not it can be written as sum of 2 squares.

(a) $3000 = 2\times2\times5\times5\times2\times\boxed{3}\times5$ CAN'T

(b) $3050 = 2\times5\times5\times\boxed{61}^{\sim 4k+1}$ CAN (Handbook)

(c) $3100 = 2\times2\times5\times5\times\boxed{31}^{\sim 4k+3}$ CAN'T

So we try to write 3050 as the sum of 2 squares.

$\underbrace{2\times5\times5}_{(7^2+1^2)}$ $\underbrace{\times 61}_{(5^2+6^2)}$

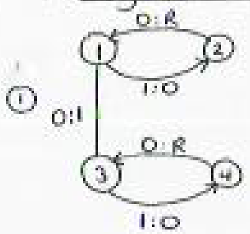
$= (7.5 + 1.6)^2 + (7.6 - 1.5)^2$

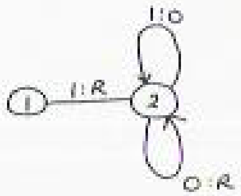
$= 41^2 + 37^2$

this uses $(a^2+b^2)(c^2+d^2) = (ac+bd)^2 + (ad-bc)^2$

(9)(i) Want a Turing machine input (m,n) out put scanning 1 on otherwise blank tape.

(a) Why are these NOT suitable?

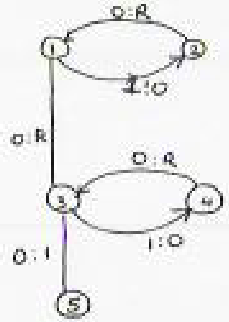
① 

② 

① Left scanning 0

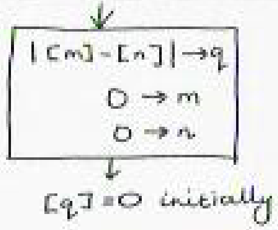
② moves to the ~~left~~ forever.

(b) Give a suitable machine.



Often a simple change will do.

(ii) Want an Abacus machine program to effect:



where $|x-y| = \begin{cases} x-y & \text{if } x \geq y \\ y-x & \text{if } x < y \end{cases}$