(12) (i) <u>Show</u> $c(x,y) = \begin{cases} 1, & x \text{ divisible by } 2^y \\ 0, & \text{otherwise} \end{cases}$ <u>is p.r.</u>

We know $d(x,y) = \begin{cases} 1 & x \text{ is div by } y \\ 0 & \text{otherwise} \end{cases}$ is p.r.

Let $d^*(x,y) = d(x, 2^y) = Cn[d, id_1^2, Cn[exp, \underbrace{2}_{2 = s(s(id_1^2))}, id_2^2]$

Thus d^* is p.r. and $d^* = c(x,y)$ as required.

(ii) <u>By summing $c(x,y)$ for appropriate values of y, or otherwise, show that</u>

$g(x)$ = greatest integer y s.t. x is div by 2^y

<u>is p.r.</u>

$g(1) = g(3) = 0$, $g(2) = g(6) = 1$, $g(4) = 2$ etc.

<u>Observe</u> If x is divisible by 2^y it is divisible by all 2^k $k < y$ also

$g(x) = c(x,0) + c(x,1) + c(x,2) \cdots\cdots \underset{}{-1}$ to remove 2^0

certainly for $y > x$ 2^y will not divide x so we will only be adding 0's

$$g(x) = \sum_{y=0}^{y=x} c(x,y) \dot{-} 1$$

(e.g. $g(4) = \underset{1}{c(4,0)} + \underset{1}{c(4,1)} + \underset{1}{c(4,2)} + \underset{0}{c(4,3)} + \underset{0}{c(4,4)} - 1$

$= 3 - 1 = 2.$

(13) Show $((\forall x\ (x = y \vee \exists z\ \ x = z) \to -(x = y \vee \exists z\, x = z)) \to$
$(-\forall x\ (x = y \vee \exists z\ \ x = z) \vee -x = y))$ <u>is a taut.</u>

Let $\phi := \forall x\ (x = y \vee \exists z\ x = z)$
$\psi := -(x = y \vee \exists z\ x = z)$
$\chi := -x = y$

We have $((\phi \xrightarrow{d} \psi) \to (-\phi \vee \chi))$

Put in truth values

(ii) <u>Fill in the assumption numbers:</u>

1	(1)	$\forall x(\phi \Leftarrow \psi)$	Ass	1
2	(2)	$(-\theta \Leftrightarrow -\psi)$	Ass	2
1	(3)	$(\phi \Leftarrow \psi)$	UE (1)	1
4	(4)	$\exists x(-\theta \Leftrightarrow -\psi)$	Ass	4
2	(5)	$(\theta \Leftrightarrow \psi)$	Taut (2)	2
1,2	(6)	$(\phi \Leftarrow \theta)$	Taut (3),(5)	1,2
1,2	(7)	$\exists x(\phi \Leftarrow \theta)$	EI (6)	1,2
1,4	(8)	$\exists x(\phi \Leftarrow \theta)$	EH (7)	1,4
4	(9)	$(\forall x(\phi \Leftarrow \psi) \to \exists x(\phi \Leftarrow \theta))$	CP, (8)	4

(b) <u>What is the taut on (6).</u>

$((\phi \Leftarrow \psi) \Leftarrow (\theta \Leftrightarrow \psi) \to (\phi \Leftarrow \theta))$

(c) <u>If we add</u> 4 (10) $(\phi \Leftrightarrow \psi)$ EH,(5)
<u>Is it still correct?</u>

No, because we have no idea if x has free occurrences in $(\phi \Leftrightarrow \psi)$

(14) (i) <u>Which of the following are freely substitutable for x in</u>
$\exists t(\forall z\ \forall \underline{x}(\underline{x} + z) = y \to \exists y\ x = y')$

(a) $(0' + 0')$ – <u>yes</u>, no variables so none becomes bound
(b) $(x \cdot t)$ – the formula would be $\exists t(\forall z\ \forall x\ (x \cdot z) = y \to \exists y\ (x \cdot t) = y'$ so <u>NO</u> becomes bound
(c) $(x \cdot z)$ – <u>yes</u> the subst^n in the only free occurrence of x doesnt bind z or x.

(ii) <u>We want a formal proof of</u>

$\forall x(\phi \Leftrightarrow -\psi) \vdash (\exists x(\psi \vee \theta) \to -\forall x(\phi \Leftarrow -\theta)$