Question 12

In this question you may use any of the recursive functions, or results about them, given in the Logic Handbook without proving that they are recursive. You may also give your answers as informal definitions.

(i) Show that the function c defined by

$$c(x,y) = \begin{cases} 1, & \text{if } y^2 \leq x, \\ 0, & \text{otherwise,} \end{cases}$$

is primitive recursive. [3]

(ii) Show that if f is primitive recursive function of 2 arguments, then the function g defined by

$$g(x,v,w) = \begin{cases} \sum_{z=v}^{w} f(x,z), & \text{if } w \geq v, \\ 0, & \text{if } w < v, \end{cases}$$

is also primitive recursive. [3]

(iii) By summing the values of $c(x,z)$ for appropriate values of z, or otherwise, show that the function k defined by

$$k(x) = \left[x^{1/2}\right],$$

i.e. the greatest integer less than or equal to the square root of x (so that e.g. $k(0) = 0$, $k(1) = k(2) = k(3) = 1$, $k(4) = \cdots = k(8) = 2$, $k(9) = 3$) is primitive recursive. [4]

(iv) Explain briefly how to adapt the above argument to show that the function j defined by

$$j(x) = [x^{1/4}],$$

i.e. the greatest integer less than or equal to the fourth root of x, is primitive recursive. [1]