Question 4

(i) Use Fermat's Little Theorem (FLT):

(a) to determine the remainder when $5^{50} + 3^{30}$ is divided by 13;

(b) to prove that

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$$

where p and q are distinct primes. [6]

(ii) You are given that the recurring decimal of $1/19$ is

$$1/19 = 0.\langle 052631578947368421 \rangle.$$

Answer the following, in each case giving brief justification of your answer. 11

(a) What is the order of 10 modulo 19?

(b) What is the value of 10^9 modulo 19?

(c) Write down the recurring decimal of $\frac{12}{19}$. [5]

Question 5

This question is concerned with the multiplicative functions σ and ϕ, where σ denotes the 'sum of the divisors' function and ϕ is Euler's ϕ-function.

(i) An integer n with $\sigma(n) > 2n$ is said to be an *abundant* number.

(a) Is 74 abundant?

(b) Is 174 abundant?

(c) For which primes p is $10p$ abundant? [7]

(ii) Show that if p and $2p - 1$ are both odd primes then $\phi(4p) = \phi(4p - 2)$. [4]

Question 6

(i) Determine whether or not the quadratic congruence

$$2x^2 - 5x + 4 \equiv 0 \pmod{37}$$

has solutions. [3]

(ii) Evaluate the Legendre symbol $(67/107)$. 11 [3]

(iii) Determine all primes $p > 3$ for which

$$(6/p) = 1.$$

[You may use results given in the Handbook relating to $(2/p)$ and $(3/p)$ if you so wish.] [5]

Question 7

(i) Determine the two simple finite continued fractions for $\frac{39}{14}$. [2]

(ii) Determine the irrational number whose continued fraction is $[2, \langle 3, 1 \rangle]$. 11 [4]

(iii) Determine the infinite simple continued fraction for $\frac{\sqrt{5}}{2}$. [5]