

Question 16

- (i) Explain briefly what is meant by saying that the theory Z (of Elementary Peano Arithmetic) is *consistent*. [2]

- (ii) (a) Give a formal proof in the theory Z to show that

$$\vdash_Z \neg 0 = 1. \quad [2]$$

- (b) Deduce that if Z is consistent then $0 = 1$ is not a theorem of Z . [1]

- (iii) Explain briefly when the formula $\text{Prov}(y)$ is true in the standard interpretation. [2]

- (iv) Which theorem(s) of the course give(s) an answer to Hilbert's Question:

Can the consistency of number theory be proved using only non-dubious principles of finitary reasoning?

Explain why the theorem(s) answer(s) the question.

(Your answer may include references to any of the theorems listed in the Logic Handbook.) [4]

[END OF QUESTION PAPER]