Question 10

(i) Let h be the function

$$\mathrm{Pr}[\mathrm{Cn}[s, z], \mathrm{Cn}[\exp, \mathrm{Cn}[s, \mathrm{id}_3^3], \mathrm{Cn}[s, \mathrm{id}_2^3]]]$$

where exp is the function defined by $\exp(x, y) = x^y$.

Compute the values of

(a) $h(2, 0)$,

(b) $h(2, 2)$. [4]

(ii) In this part you may present your arguments using either formal or informal definitions.

(a) Show that the function exp, as in part (i), is primitive recursive by defining it in terms of the sum function. [2]

(b) Show that the function f of 3 arguments defined by

$$f(x_1, x_2, x_3) = x_1^{(x_3^{x_2})}$$

is primitive recursive by defining it in terms of exp and, if you wish, the sum and product functions. [2]

(iii) For which pairs (x_1, x_2) of natural numbers is $\mathrm{Mn}[f](x_1, x_2)$ defined, where f is the function defined by

$$f(x_1, x_2, y) = (x_2 \mathbin{\dot{-}} (y + x_1)) + (x_1 \mathbin{\dot{-}} 7) \cdot y?$$ [3]

Question 11

(i) A Turing machine has a configuration with left number 41 and right number 91. Draw the configuration which results when the scanning head has moved one square to the right, and find its left and right numbers. [3]

(ii) In parts (a), (b) and (c) below, you may use any of the recursive functions except e, Eq and d, or results about recursive functions, given in the Logic Handbook without proving that they are recursive.

(a) Show that the condition "$x = y$" on pairs (x, y) of natural numbers is primitive recursive. [$2\frac{1}{2}$]

(b) Show that the condition "x is even" on natural numbers x is primitive recursive. [2]

(c) Show that the function f defined by

$$f(x, y) = \begin{cases} x^{3y}, & \text{if } x \cdot y + 4 \text{ is odd,} \\ y, & \text{if } 2x + 3y = 8000, \\ 2, & \text{otherwise,} \end{cases}$$

is primitive recursive. [$3\frac{1}{2}$]