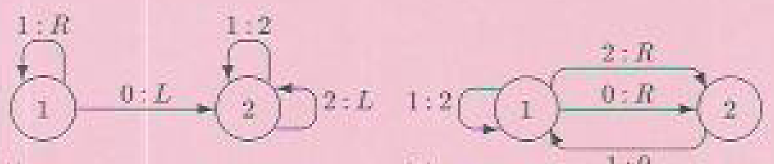
Question 9

(i) We wish to design a Turing machine which, if started scanning the leftmost of a string of n 1s (on an otherwise blank tape), would halt scanning a single 2 on an otherwise blank tape.

(a) Explain why each of the Turing machines below is *not* suitable for this task. (Your answer may include sequences of configurations for appropriate test data.)



(1) (2)

[4]

(b) Give the flowgraph of a machine which correctly performs the task. [2]

(ii) Give the complete flowchart of an Abacus machine program which has the effect shown in the following block diagram. (You may use extra registers, assumed empty initially, if you wish.)

$$\downarrow$$
$$\boxed{\begin{array}{r} \mathrm{Max}([m],[n]) \longrightarrow p \\ |[m]-[n]| \longrightarrow n \\ 0 \longrightarrow m \end{array}}$$
$$\downarrow$$

where $[p] = 0$ initially.

where $\mathrm{Max}(x,y) = \begin{cases} y, & \text{if } y \geq x, \\ x, & \text{if } y < x, \end{cases}$

and $|x-y| = \begin{cases} y-x, & \text{if } y \geq x \\ x-y, & \text{if } y < x. \end{cases}$

[5]

Question 10

(i) Let h be the function

$$\mathrm{Pr}[s, \mathrm{Cn}[\mathrm{prod}, \mathrm{Cn}[s, \mathrm{id}_3^3], \mathrm{Cn}[s, \mathrm{id}_2^3]]]$$

where prod is the product function defined by $\mathrm{prod}(x,y) = x \cdot y$.

Compute the values of

(a) $h(4,0)$,

(b) $h(4,2)$. [4]

(ii) In this part you may present your arguments using either formal or informal definitions.

(a) Show that the function prod, as in part (i), is primitive recursive by defining it in terms of the initial functions. [2]

(b) Show that the function f of 3 arguments defined by

$$f(x_1, x_2, x_3) = (x_1 \cdot x_2)^{x_3}$$

is primitive recursive by defining it in terms of the initial functions and, if you wish, the sum and product functions. [2]

(iii) Write down the pairs (x_1, x_2) of natural numbers for which $\mathrm{Mn}[f](x_1, x_2)$ is defined, where f is the function defined by

$$f(x_1, x_2, y) = x_1^{y+1} \cdot ((x_2 + y) \dot{-} x_1).$$ [3]