Question 15

For each of the following sentences, decide whether or not it is a theorem of Q. If it is a theorem of Q, write down a formal proof showing this. If it is not a theorem of Q, justify this. (*You may use without proof the fact that all the axioms of Q are true under the interpretations N^* and N^{**} given in the Logic Handbook.*)

(i) $\forall x((\mathbf{0} \cdot \mathbf{0}) \cdot x) = ((\mathbf{0} \cdot x)) + (x \cdot \mathbf{0}))$

(ii) $\forall x \forall y(x' \cdot y) = ((x \cdot y) + y)$

(iii) $\forall x \exists y(x' \cdot y) = (y' \cdot x)$ [*11*]

Question 16

(i) Explain briefly what is meant by Church's Thesis, what evidence there is for it to be true and what would be needed to show it false. [*3*]

(ii) Give brief explanations of *each* of the following:

(a) a *decidable* theory;

(b) the theory *arithmetic*. [*3*]

(iii) Is the theory Z (of Elementary Peano Arithmetic) decidable? Briefly explain your answer. [*2*]

(iv) Which theorem (or theorems) of the course give(s) an answer to Leibniz's Question:

Is there an algorithm for deciding which statements of number theory are true?

Explain why the theorem(s) answer(s) the question. In particular, what roles, if any, do Church's Thesis and the theory arithmetic play in answering the question? [*3*]

(*Your answer may include references to any of the theorems listed in the Logic Handbook.*)

[END OF QUESTION PAPER]