Question 11

(i) A Turing machine has a configuration with left number 29 and right number 54. Draw the configuration which results when the scanning head has moved one square to the left, and find its left and right numbers. [3]

(ii) In parts (a), (b) and (c) below, you may use any of the recursive functions, or results about them, given in the Logic Handbook without proving that they are recursive. You may give your answers as informal definitions.

(a) Show that the condition "$y \geq x$" on pairs (x, y) of natural numbers is primitive recursive. [3]

(b) Show that the function Max defined by

$$\text{Max}(x, y) = \begin{cases} y, & \text{if } y \geq x, \\ x, & \text{if } y < x, \end{cases}$$

is primitive recursive. [$1\frac{1}{2}$]

(c) Show that the function f defined by

$$f(x_1, x_2, x_3) = \begin{cases} x_1^{x_2}, & \text{Max}(x_1, x_3) \geq x_2 + 21, \\ 7, & \text{if } 3x_1 + 4x_2 + 5x_3 = 60, \\ 3x_3, & \text{otherwise}, \end{cases}$$

is primitive recursive. [$3\frac{1}{2}$]

Question 12

In this question, with the exception of the proviso given in part (i), you may use any of the recursive functions, or results about them, given in the Logic Handbook without proving that they are recursive. You may also give your answers as formal or informal definitions.

(i) Prove that if f is a primitive recursive function of 2 arguments, then the function h defined by

$$h(x, y) = \sum_{z=0}^{y} f(x, z)$$

is also primitive recursive. You may *not* use the result in the Handbook of which this is a special case. [3]

(ii) By summing the values of $d(x, y)$ for appropriate values of y, where the function d (given as primitive recursive in the Logic Handbook) is defined by

$$d(x, y) = \begin{cases} 1, & \text{if } x \text{ is divisible by } y, \\ 0, & \text{otherwise}, \end{cases}$$

show that the function p defined by

$$p(x) = \begin{cases} 1, & \text{if } x \text{ is a prime number}, \\ 0, & \text{otherwise}, \end{cases}$$

is primitive recursive. [5]

(iii) Show that the function t defined by

$$t(x) = \begin{cases} 1, & \text{if } x \text{ and } x + 2 \text{ are both prime}, \\ 0, & \text{otherwise}, \end{cases}$$

is primitive recursive. [3]