Question 13

(i) Show that the following formula takes truth value 1 under all interpretations of its symbols.

$$((-\exists y\ y = z \longrightarrow (-y = z\ \&\ \exists y(y = z \vee \exists y\ y = z))) \longrightarrow (-y = z \vee \exists y\ y = z))$$ [3]

(ii) The following is a correct (but contorted) proof from which the assumption numbers have been omitted.

(1)	$\forall x(\theta\ \&\ -\phi)$	Ass	1
(2)	$(\theta\ \&\ -\phi)$	UE, (1)	1
(3)	$(\phi \rightarrow \psi)$	Ass	3
(4)	$\exists x(\phi \rightarrow \psi)$	Ass	4
(5)	$(\phi \rightarrow \psi)$	Taut, (2), (3)	1, 3
(6)	$(\phi \rightarrow \psi)$	Taut, (2)	1
(7)	$\exists x(\phi \rightarrow \psi)$	EI, (5)	1, 3
(8)	$\exists x(\phi \rightarrow \psi)$	EH, (7)	1, 4
(9)	$(\exists x(\phi \rightarrow \psi) \rightarrow \exists x(\phi \rightarrow \psi))$	CP, (8)	1,

(a) Write down the assumptions in force on each line. [$2\frac{1}{2}$]

(b) Write down the tautology used on line (5). [$\frac{1}{2}$]

(c) For each of the following lines, write down whether the proof would still be correct were the line to be added to it.

(A)	3	(10)	$\forall x(\phi \rightarrow \psi)$	UI, (3)
(B)	1	(10)	$\exists x(\phi \rightarrow \psi)$	EI, (6)

Answer YES or NO. [2]

(iii) The rule UI states that:

if the formula ϕ occurs on a line of a formal proof and <u>the variable v does not occur free in any of the assumptions in force on this line</u>, then on any subsequent line we may derive $\forall v\phi$, and this formula will depend on the same assumptions as did ϕ.

Give a suitable example from every day mathematics to show that the rule is no longer valid if the underlined passage, which gives a condition on the variable v, is omitted. [3]

Question 14

(i) Which of the following terms are freely substitutable for x in the formula

$$\forall y(\exists x \forall t(x + t) = z\ \&\ \forall z(x + z) = t)$$

(a) $(x \cdot z)$

(b) $(0 + y)$

(c) $(t + x)$

Answer YES or NO in each case. [2]

(ii) Give formal proofs to establish each of the following results.

(a) $\exists x\ x' = x \vdash \exists y\ y' = y$ [3]

(b) $\forall x\theta \vdash (-\phi \rightarrow -\exists x(\theta \rightarrow \phi))$, where the variable x does not occur free in ϕ. Indicate the step(s) of your proof which require the condition on ϕ. [6]