Question 12

In this question you may use any of the recursive functions, or results about them, given in the Logic Handbook without proving that they are recursive. You may also give your answers as informal definitions.

(i) Show that the function c defined by

$$c(x, y, z) = \begin{cases} 1, & \text{if } yz \leq x, \\ 0, & \text{otherwise,} \end{cases}$$

is primitive recursive. [3]

(ii) The result of the Euclidean algorithm says that, given two natural numbers x, y with $y > 0$, there exist unique natural numbers q and r such that

$$x = qy + r \text{ with } 0 \leq r < y.$$

The numbers q and r are called respectively the *quotient* and *remainder* on division of x by y.

(a) By summing the values of $c(x, y, z)$ for appropriate values of z, or otherwise, show that the function quot defined by

$$\text{quot}(x, y) = \text{ the quotient on division of } x \text{ by } y \text{ (where } y \neq 0)$$

is primitive recursive. [6]

(b) Show that the function rem defined by

$$\text{rem}(x, y) = \text{ the remainder on division of } x \text{ by } y \text{ (where } y \neq 0)$$

is primitive recursive. [2]