Question 2

Assume that

$$Q = kh^{\alpha}g^{\beta}\theta^{\gamma}U^{\delta},$$

where k is a dimensionless constant.

Then $[Q] = [h]^{\alpha}[g]^{\beta}[\theta]^{\gamma}[U]^{\delta}$,

i.e. $\mathrm{L}^3\mathrm{T}^{-1} = \mathrm{L}^{\alpha}\,\mathrm{L}^{\beta}\,\mathrm{T}^{-2\beta}\,\mathrm{L}^{\delta}\,\mathrm{T}^{-\delta}$, since $[\theta] = 1$ (see page 16 of the Handbook). $2\frac{1}{2}$

Thus,

$$\text{L:}\quad 3 = \alpha + \beta + \delta,$$
$$\text{T:}\quad -1 = -2\beta - \delta.$$

Let $\delta = a$. Then

$$\beta = \frac{1-a}{2},$$
$$\alpha = 3 - \frac{1-a}{2} - a = \frac{5}{2} - \frac{a}{2}.$$ 2

So

$$\begin{aligned} Q &= kh^{(5-a)/2}g^{(1-a)/2}\theta^{\gamma}U^{a} \\ &= kh^{5/2}g^{1/2}\left(\frac{U}{(gh)^{1/2}}\right)^{a}\theta^{\gamma}. \end{aligned}$$

More generally, Q can be written in the form $g^{1/2}h^{5/2}f\left(\dfrac{U}{(gh)^{1/2}},\theta\right)$, where f is an undetermined function. $1\frac{1}{2}$

[6]

Question 3

(i) From page 18 of the Handbook, or the Bookmark,

$$\begin{aligned} \operatorname{div}\mathbf{u} &= \frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{\partial u_z}{\partial z} \\ &= \frac{1}{r}\frac{\partial}{\partial r}(re^{\theta}) + \frac{1}{r}\frac{\partial}{\partial\theta}(-e^{\theta}) + 0 \\ &= \frac{e^{\theta}}{r} - \frac{e^{\theta}}{r} = 0, \end{aligned}$$

so flow is incompressible. $1\frac{1}{2}$

(ii) From page 19 of the Handbook, or the Bookmark,

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = e^{\theta}, \qquad (1)$$
$$\frac{\partial\psi}{\partial r} = e^{\theta}. \qquad (2)$$

From (2), $\psi = re^{\theta} + f(\theta)$, so

$$\frac{\partial\psi}{\partial\theta} = re^{\theta} + f'(\theta). \qquad (3)$$

From (1) and (3), $f'(\theta) = 0$, and so $f(\theta) =$ constant.

Thus $\psi(r,\theta) = re^{\theta}+$ constant.

The equation of the streamlines is $3\frac{1}{2}$

$$re^{\theta} = \text{constant}.$$