(v) $u_z = -\frac{p_0 - p_1}{4l\mu} r^2 + A \log_e r + B,$

$$\frac{du_z}{dr} = -\frac{p_0 - p_1}{2l\mu} r + \frac{A}{r},$$

$$\frac{d^2 u_z}{dr^2} = -\frac{p_0 - p_1}{2l\mu} - \frac{A}{r^2}.$$

Therefore

$$\frac{d^2 u_z}{dr^2} + \frac{1}{r}\frac{du_z}{dr} = -\frac{p_0 - p_1}{2l\mu} - \frac{A}{r^2} - \frac{p_0 - p_1}{2l\mu} + \frac{A}{r^2}$$
$$= \frac{p_1 - p_0}{l\mu},$$

and so the equation is satisfied.

Now $u_z = 0$ when $r = a$, $r = 2a$, so

$$0 = -\frac{(p_0 - p_1)}{4l\mu} a^2 + A \log_e a + B,$$

$$0 = -\frac{(p_0 - p_1)}{l\mu} a^2 + A \log_e 2a + B.$$

Subtracting gives $0 = \frac{(p_0 - p_1)3a^2}{4l\mu} - A \log_e 2$, therefore

$$A = \frac{(p_0 - p_1)3a^2}{4l\mu \log_e 2}.$$

Also,

$$B = \frac{(p_0 - p_1)}{4l\mu} a^2 - A \log_e a$$
$$= \frac{(p_0 - p_1)}{4l\mu} a^2 - \frac{(p_0 - p_1)3a^2 \log_e a}{4l\mu \log_e 2}, \quad \text{on substituting for } A.$$ 5

[20]

---

**Question 10**

(i) $|\sin x|$ is even, so the Fourier series for $|\sin x|$ is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx,$$

where $a_n = \frac{2}{\pi} \int_0^{\pi} \sin x \cos nx \, dx$

$$= \frac{1}{\pi} \int_0^{\pi} [\sin(1+n)x + \sin(1-n)x] \, dx$$

$$= \frac{1}{\pi} \left[ -\frac{1}{1+n} \cos(1+n)x - \frac{1}{1-n} \cos(1-n)x \right]_0^{\pi} \quad (n \neq 1).$$

For $n = 1$, $a_1 = \frac{1}{\pi} \left[ -\frac{\cos 2x}{2} \right]_0^{\pi}$. 4

On substituting the limits and simplifying, we have

$a_n = 0$, if $n$ is odd,

$a_n = \frac{4}{\pi(1 - n^2)}$, if $n$ is even;

therefore the Fourier series for $|\sin x|$ is

$$\frac{2}{\pi} + \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{\cos 2mx}{1 - 4m^2}.$$ 2

*Solution to Question 10 continued overleaf*