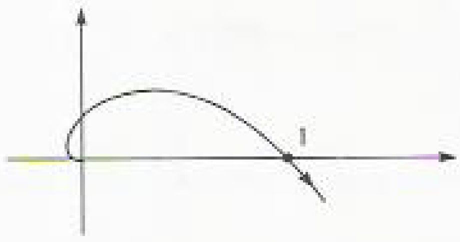
(iii) When $r = 1$ and $\theta = 0$, the constant is 1.

The required streamline is $re^{\theta} = 1$. 1



1

(Note $\mathbf{u}$ has same magnitude ($e^{\theta} = \frac{1}{r}$) in the $\mathbf{e}_r$ and $-\mathbf{e}_\theta$ directions along this streamline which can be identified as one for a combination of a source and a negative vortex at the origin.)

[7]

Question 4

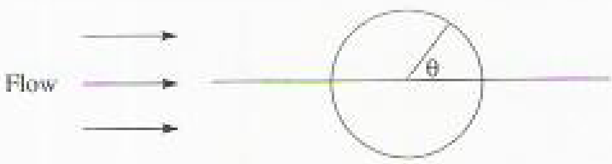
$$\psi = U\left(r - \frac{a^2}{r}\right)\sin\theta + \frac{\kappa}{2\pi}\log_e\left(\frac{r}{a}\right)$$

(i) From page 19 of the Handbook,

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta,$$

$$u_\theta = -\frac{\partial\psi}{\partial r} = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta - \frac{\kappa}{2\pi r}.$$ 2

(ii)



On the cylinder,

$$r = a, \quad u_r = 0 \quad \text{and} \quad u_\theta = -2U\sin\theta - \frac{\kappa}{2\pi a}.$$ 1

A typical cross-sectional circle $r = a$, $z =$ constant, is a streamline. The fluid is incompressible and inviscid, there is no body force and the flow is steady, so we can use Bernoulli's equation along this streamline,

i.e. $p + \frac{1}{2}\rho u_\theta^2 =$ constant.

Therefore

$$p + \tfrac{1}{2}\rho\left(2U\sin\theta + \frac{\kappa}{2\pi a}\right)^2 = M \quad \text{(a constant)},$$

so $$p = M - \tfrac{1}{2}\rho\left(4U^2\sin^2\theta + \frac{2U\kappa\sin\theta}{\pi a} + \frac{\kappa^2}{4\pi^2 a^2}\right).$$

From page 22 of the Handbook,

$$\begin{aligned}
\text{lift/unit length} &= \int_0^{2\pi}(-p\sin\theta)a\,d\theta \\
&= -\int_0^{2\pi}\left[M - \frac{1}{2}\rho\left(4U^2\sin^2\theta + \frac{2U\kappa\sin\theta}{\pi a} + \frac{\kappa^2}{4\pi^2a^2}\right)\right]a\sin\theta\,d\theta \\
&= \frac{\rho\kappa U}{\pi}\int_0^{2\pi}\sin^2\theta\,d\theta + 0 + 0 + 0 \\
&= \frac{\rho\kappa U}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \rho U\kappa.
\end{aligned}$$ 4

[7]
