Question 7

As with all essay-type questions there will be a variety of answers and the marking scheme is flexible as a result. With reference to the five equations mentioned in the question, groups of four marks could be awarded for:

(i) verbal description;

(ii) mathematical description;

(iii) indication of how related to Navier–Stokes equation;

(iv) application/modelling.

Of course, any valid point scores a mark but not an additional mark if the same point is repeated. Some suggested answers follow (in note form).

Navier–Stokes equation

Modelling viscous fluid flow (in conjunction with continuity equation)

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\,\mathbf{u}\right) = -\boldsymbol{\nabla}p + \rho\mathbf{F} + \mu\nabla^2\mathbf{u}. \qquad \text{(Handbook, page 23)}$$

Invariant form

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\nabla}(\tfrac{1}{2}u^2) - \mathbf{u}\times(\boldsymbol{\nabla}\times\mathbf{u})\right) = -\boldsymbol{\nabla}p + \rho\mathbf{F} - \mu\Big(\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{u})\Big).$$

Could quote in cylindrical/spherical coordinates and argue that certain geometries lend themselves to these forms.

Euler's equation

Inviscid $\mu = 0$.
Applicable away from boundaries, for 'mainstream' flow

$$\rho\frac{d\mathbf{u}}{dt} = \rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}\right) = -\boldsymbol{\nabla}p + \rho\mathbf{F}.$$

Bernoulli's equation

Integral form of Euler's equation $\therefore$ inviscid.

For $\mathbf{F} = \boldsymbol{\nabla}\Omega$ (conservative body forces) and steady and irrotational flows, of constant density,

$$\boldsymbol{\nabla}\left(\tfrac{1}{2}u^2\right) - \mathbf{u}\times(\boldsymbol{\nabla}\times\mathbf{u}) = -\frac{1}{\rho}\boldsymbol{\nabla}p + \boldsymbol{\nabla}\Omega,$$

$\therefore\ \frac{1}{2}u^2 + p - \Omega =$ constant along any curve in fluid

(or any or all of 3 forms given in Handbook *plus* conditions under which they hold).

High (low) pressure $\leftrightarrow$ low (high) speed.

Flow along tubes, channels.

Equation of creeping motion

$$\boldsymbol{\nabla}p = \mu\nabla^2\mathbf{u}$$

$\rho\dfrac{d\mathbf{u}}{dt}$ (inertial terms) neglected and body force neglected. Small Re flow.

Balance between pressure and viscous forces. Bearings, roller coating, wire coating