

MST322 Solutions to the Specimen Examination Paper

This is a rough guide to the type of written solutions required. We do not expect your solutions to be exactly like these and, of course, for some questions there may be alternative ways of doing them. Any correct method receives full marks.

A mark scheme is provided so that you have some indication of how marks will be allocated.

Solutions to Part I

Question 1

(i) $x = 0$ 1

(ii) Put $y = x^\lambda$.

The resulting indicial equation is

$$\lambda^2 + \lambda + 1 = 0.$$

Thus, $\lambda = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$. 2

The general solution is (see page 14 of the Handbook)

$$y = x^{-\frac{1}{2}} \left[A \cos \left(\frac{\sqrt{3}}{2} \log_e x \right) + B \sin \left(\frac{\sqrt{3}}{2} \log_e x \right) \right] \quad (x > 0),$$

where A and B are constants. 1

(iii) Put $z = -x$ so that $z > 0$ when $x < 0$.

The differential equation becomes

$$z^2 \frac{d^2 y}{dz^2} + 2(-z) \left(-\frac{dy}{dz} \right) + y = 0$$

or $z^2 \frac{d^2 y}{dz^2} + 2z \frac{dy}{dz} + y = 0.$

This equation has general solution (by part (ii)) for $z > 0$ of the form

$$y = z^{-\frac{1}{2}} \left[A \cos \left(\frac{\sqrt{3}}{2} \log_e z \right) + B \sin \left(\frac{\sqrt{3}}{2} \log_e z \right) \right]$$

$$= (-x)^{-\frac{1}{2}} \left[A \cos \left(\frac{\sqrt{3}}{2} \log_e (-x) \right) + B \sin \left(\frac{\sqrt{3}}{2} \log_e (-x) \right) \right]$$

$(x < 0).$ 2

[6]