Question 11

(i) Consider the velocity potential

$$\phi(x, z, t) = 2e^{kz} \sin(kx - \omega t), \tag{3}$$

in which x and z are Cartesian coordinates, t represents time, and ω and k are positive constants.

(a) Find the velocity vector field associated with this potential, and show that the flow represented by this vector field satisfies the continuity equation for the flow of an incompressible fluid. [2]

(b) Write down the three equations that must be satisfied by waves in an inviscid, incompressible fluid of *infinite* depth with free surface $z = 0$ and show that these equations are satisfied provided that $\omega^2 = gk$ where g is the magnitude of the acceleration due to gravity. [5]

(c) Consider now the waves in the free surface $z = 0$. Determine the surface profile of these waves. Are these progressive waves? Are they dispersive? What is the amplitude of these waves? [5]

(ii) A composite wave is the sum of two individual waves given by:

$$z_1 = \cos 1.9(x - 2t) \quad \text{and} \quad z_2 = \cos 2.1(x - 2t).$$

What are the equations of the wave envelope? For $t = 0$, at which points within the wave envelope is the wave profile zero and at which points does it touch the envelope? What are the wavelength and the period of the composite wave? State whether or not the wave profile moves relative to the envelope, giving a brief reason for your answer. [8]

[END OF QUESTION PAPER]