Question 5

Inviscid fluid of constant density ρ flows past a fixed impermeable sphere of radius a. The flow at large distances upstream and downstream of the sphere is uniform with speed U and pressure P. The velocity potential has the form

$$U\cos\theta\left(r+\frac{1}{2}\frac{a^3}{r^2}\right).$$

Indicate on a diagram how r and θ are related to the flow. Show that the boundary conditions for the flow at the sphere, and at large distances, are satisfied by the given velocity potential. Determine the pressure distribution over the surface of the sphere. Show your working. [6]

Question 6

Consider the regular Sturm–Liouville problem

$$[(3x+1)^{\frac{1}{3}}y']' - 2(3x+1)^{-\frac{2}{3}}y + \lambda(3x+1)^{-\frac{2}{3}}y = 0 \qquad (0<x<1)$$
$$y(0)=0,\ y(1)=0,$$

where $y=y(x)$.

(i) What is the condition of the Comparison Theorem for Eigenvalues for this problem? (You are *not* asked to show that this condition is true.)

(ii) Use the Comparison Theorem for Eigenvalues to find upper and lower bounds for each eigenvalue λ_n $(n=1,2,\ldots)$ of the problem. [8]