PART II

Answer **THREE** *questions in this part.*

Each question carries 20% of the total examination marks.

Question 7

(i) Imagine a fictitious situation in which Euler, Poiseuille and Bernoulli had an opportunity to meet to discuss the work attributed to them in MST322. Give a *brief* account of the fluid properties (including equations) which each might have put forward to support his results and theory of fluid flows, as described in the course units. *[10]*

(ii) Write down the Navier–Stokes and continuity equations in cylindrical polar coordinates (r, θ, z) in the case where there are no body forces. Show, clearly, that these equations can be simplified for a cylindrically symmetric, steady, incompressible flow with velocity vector, $\mathbf{u} = (u_r, u_\theta, u_z)$, in which $u_r = u_\theta = 0$, to:

$$0 = -\frac{dp}{dz} + \mu\left(\frac{d^2u_z}{dr^2} + \frac{1}{r}\frac{du_z}{dr}\right), \tag{1}$$

explaining, carefully, why the pressure p and the velocity component u_z are, respectively, functions of z and r alone. State what the remaining symbol μ in Equation (1) represents. Hence solve Equation (1) for u_z, in the case where $dp/dz = 0$, giving your answer in terms of two unknown constants. *[10]*

Question 8

(i) Consider a two-dimensional flow whose velocity vector field $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ at time t has Cartesian components

$$u_1 = -2x + 2t \quad \text{and} \quad u_2 = 2y \quad (x > 0, y > 0).$$

Is the flow

(a) incompressible,

(b) irrotational,

(c) steady?

Give reasons for your three answers. [4]

(ii) Write down the equations describing the stream function for the velocity vector field in Part (i). Hence find the stream function for this flow. Sketch some of the streamlines at $t = 0$, showing the direction of flow. [8]

(iii) Consider the flow of an inviscid fluid of constant density ρ given by the velocity vector field of Part (i) with body force (per unit mass)

$$\mathbf{F} = 4y\mathbf{j}.$$

Find the pressure distribution in the fluid (to within an arbitrary function of time). [8]