Part I

Answer **ALL SIX** *questions in this part.*

The questions in this part are not all worth the same number of marks.

The number of marks assigned to each question is given in square brackets.

Part I as a whole carries 40% of the total examination marks.

Question 1

An atmosphere of static air occupies the region $z \geq 0$ and is modelled as a perfect gas under isothermal conditions. Show that the pressure at any height z is given by

$$p = p_0 \exp(-z/H),$$

where p_0 is the pressure at $z = 0$, and H is a constant. Determine an expression for H in terms of ρ_0, p_0 and g, where g is the magnitude of the acceleration due to gravity.

A rigid sphere containing gas of pressure p_0 is towed up into the atmosphere until its highest point is at height $z = H$. What is the pressure difference across the wall of the sphere at its highest point? [6]

Question 2

Solve the following problems.

(i) $x^2u''(x) + xu'(x) - 9u(x) = 0$,

$\lim_{x \to 0} u(x)$ is bounded, $u(1) = 1$.

(ii) $u''(x) - 9u(x) = 0$,

$u(0) = 2e$, $u'(0) = 0$. [7]

Question 3

Find the solution of the equation

$$\frac{\partial u}{\partial x}(x, y) - 2u(x, y) = e^{2x},$$

which satisfies $u(x, y) = y$ on $x = 0$. [6]

Question 4

The lift force on an aerofoil in a flow of speed u depends upon this speed, the surface area S of the aerofoil, and upon the angle of attack α. This force also depends on the density and coefficient of viscosity of the fluid, ρ and μ, respectively. Use the method of dimensional analysis to show that one possible formula for the magnitude F of the lift force is given by

$$F = Su^2\rho f\left(\alpha, \frac{\mu}{u\rho\sqrt{S}}\right),$$

where f is an undetermined function of two variables. [7]