Question 11

(i) Show that the Fourier cosine series for the function

$$f(x) = \sin x$$

on the interval $0 < x < \pi$ is

$$\frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}.$$

[7]

(ii) Show that the eigenvalue problem

$$X''(x) + \lambda X(x) = 0 \quad (0 < x < \pi),$$
$$X'(0) = X'(\pi) = 0$$

has eigenvalues $\lambda_n = n^2 \quad (n = 0, 1, 2, \ldots)$ and corresponding eigenfunctions $X_n(x) = \cos nx$.

[4]

(iii) The equation governing the temperature distribution $u(x,t)$ in an insulated bar of length π is given by the diffusion equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad (0 < x < \pi, t > 0),$$

where x and t represent distance along the bar and time respectively and k is a positive constant. Initially the temperature distribution of the bar is

$$u(x, 0) = \sin x \quad (0 < x < \pi).$$

The two ends of the bar are insulated (so that there is no temperature change across the ends).

(a) Write down the boundary conditions at $x = 0$ and $x = \pi$ for the temperature distribution.

(b) Use the method of separation of variables to determine the temperature distribution $u(x,t)$ at times $t > 0$.

[9]

[END OF QUESTION PAPER]