PART II

Answer **THREE** *questions in this part.*

Each question carries 20% of the total examination marks.

Question 7

(i) The velocity vector field at time $t = 0$ for a fluid is

$$\mathbf{v} = (yz + 2xz)\mathbf{i} + xz\mathbf{j} + (x^2 + xy)\mathbf{k}.$$

Show that $\mathbf{v}$ is irrotational. Find the velocity potential ϕ corresponding to $\mathbf{v}$.

Hence find $\int_A^B \mathbf{v} \cdot d\mathbf{r}$, where the line integral is along *any* path connecting the points $A = (1, 0, 1)$ and $B = (1, 1, 0)$.

What additional information about the fluid would you require to deduce that the circulation is zero around *all* closed curves at *all* times $t > 0$? [12]

(ii) The stream function of a two-dimensional, inviscid, incompressible fluid flow is given in a cylindrical polar coordinate system (r, θ, z) by

$$\psi = -a\theta - \frac{br^2}{2},$$

where a and b are non-zero constants. Evaluate $\nabla\psi$ and hence show that the velocity, given by $\nabla\psi \times \mathbf{e}_z$, where $\mathbf{e}_z$ is a unit vector perpendicular to the plane of the flow, is a superposition of two basic flows. Identify these two flows.

Show that the flow has non-zero vorticity. [8]

Question 8

(i) (a) An incompressible, inviscid fluid flows at a depth of 1 m along a horizontal, open channel of rectangular cross-section with a speed of $2\,\mathrm{m\,s^{-1}}$. Calculate the volume flow rate (per unit width), Q, and the value of the specific energy, E, for this flow.

Starting with these values of Q and E, show that values of the fluid's depth, h, must satisfy

$$5h^3 - 6h^2 + 1 = 0$$

and find any other values of h and of flow speed, u, that are possible for this channel flow.

(Take the magnitude of the acceleration due to gravity as $10\,\mathrm{m\,s^{-2}}$. As a hint, remember that $h = 1$ m will be one root of the equation for h.)

(b) What are the values of the Froude number for each of the possible flows in the channel? Hence, classify these flows as 'shallow and fast' or 'deep and slow'. [11]

(ii) (a) Discuss the following statement which appears in the MST322 Handbook.

> As well as providing a quantitative method for solving fluid flow problems, Bernoulli's equation can be used to explain certain flow phenomena using the qualitative law:
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> 'high(low) speed implies low(high) pressure'.

(b) Illustrate the qualitative law with an explanation of why rail travellers are told to stand well back from the edge of a station platform when a high-speed train passes. [9]