

Part I

Answer ALL SIX questions in this part.

The questions in this part are not all worth the same number of marks.

The number of marks assigned to each question is given in square brackets.

Part I as a whole carries 40% of the total examination marks.

Question 1

- (i) Solve the following problem:

$$x^2 u''(x) - xu'(x) + u(x) = 0 \quad (x > 0),$$
$$u(1) = 0, \quad u(e) = e.$$

- (ii) Find the general solution $u = u(x, t)$ of the equation

$$x \frac{\partial u}{\partial x}(x, t) - u(x, t) = 0 \quad (x > 0). \quad [5]$$

Question 2

The magnitude, F , of the force exerted by a ship's propeller depends on its diameter, d , and on the number of revolutions it makes per second, n . The force also depends on the speed, v , of the ship and the density, ρ , and the coefficient of viscosity, μ , of the seawater through which the ship moves.

Use the method of dimensional analysis to show that F may be expressed as

$$F = \rho d^2 v^2 f\left(\frac{\mu}{\rho v d}, \frac{dn}{v}\right),$$

where f is an undetermined function of two variables.

(Hint: consider the powers of n and μ as independent variables and choose to express the powers of the other variables in terms of them.)

[8]

Question 3

Consider the steady velocity vector field

$$\mathbf{u} = \frac{\sin 2\theta}{r^2} \mathbf{e}_r - \frac{\cos 2\theta}{2r^2} \mathbf{e}_\theta \quad (r \neq 0),$$

where r and θ are plane cylindrical polar coordinates.

- (i) Show that \mathbf{u} represents the velocity field of an incompressible flow.
- (ii) Write down the equations satisfied by the stream function for the velocity field \mathbf{u} . Hence find the stream function $\psi(r, \theta)$ for this flow and the equation of the streamlines.
- (iii) Find the equation of the pathline which passes through the point $r = 1, \theta = 0$.

[7]