Question 9

A viscous fluid of constant density ρ and constant coefficient of viscosity μ flows in the channel formed by two infinite, parallel planes, illustrated as $z = 0$ and $z = a$ of a Cartesian coordinate (x, y, z) system in Figure 1.

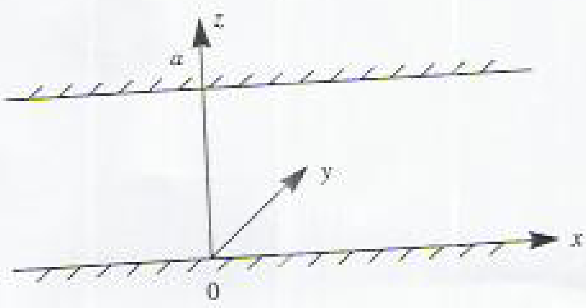

Figure 1

The plane $z = 0$ is stationary whereas the plane $z = a$ moves parallel to itself in the direction of the positive x-axis with speed Ue^{-t}, where U is a positive constant and t represents time.

(i) If the pressure in the fluid is independent of y and gravitational forces on the fluid are negligible, why is it reasonable to consider two-dimensional flow with no variation in the y direction? Write down the mathematical consequences of (a) two-dimensional flow, (b) no variation in the y direction and (c) the velocity field not changing with position x along the channel.

Hence write down the appropriate continuity equation for the fluid velocity $\mathbf{u}$, and give the boundary conditions on $\mathbf{u}$ at the upper and lower planes. Hence show that $\mathbf{u} = u_1\mathbf{i}$, where u_1 is a function of z and t only. [9]

(ii) Write down the x- and z-components of the Navier–Stokes equations for the flow and hence show that the pressure, p, is a function of x and t only. By assuming forms for u_1 and p, given by

$$u_1(z, t) = e^{-t} f(z) \quad \text{and} \quad p(x, t) = e^{-t} h(x),$$

show that the Navier–Stokes equations have a solution in which

$$f(z) = A\cos\sqrt{\frac{\rho}{\mu}}z + B\sin\sqrt{\frac{\rho}{\mu}}z - \frac{C}{\rho},$$

where A, B and C are constants. Determine expressions for A and B in terms of C, ρ, μ, U and a. [9]

(iii) Sketch the velocity profile at $t = 0$ in the case when $C = 0$. [2]