

Question 5

A rectangular channel of width 3.0 m consists of two horizontal sections joined by a gradual slope so that the bottom of the downstream section is 0.06 m higher than that of the upstream section. The flow of water in the channel is steady and is at a depth of 0.09 m in the upstream section. The water surface rises over the raised section so that the depth of water in the downstream section is 0.12 m.

Assuming that water is an inviscid, incompressible fluid, determine the volume flow rate along the channel.

[Take the magnitude of the acceleration due to gravity as 10 m s^{-2} .]

[6]

Question 6

- (i) The velocity vector field at time $t = 0$ for a fluid is

$$\mathbf{v} = (x^2 + y^2)\mathbf{i} + 2xy\mathbf{j}.$$

A closed path C is formed from the straight lines joining $O(0, 0)$, $A(1, 0)$ and $B(1, 1)$. These lines are given as follows:

$$OA: y = 0 \text{ and } x = \alpha \quad (0 \leq \alpha \leq 1)$$

$$AB: x = 1 \text{ and } y = \beta \quad (0 \leq \beta \leq 1)$$

$$\text{and } OB: x = y = \gamma \quad (0 \leq \gamma \leq 1).$$

By evaluating the line integral, show that the circulation $\oint_C \mathbf{v} \cdot d\mathbf{r}$ is equal to zero.

- (ii) Show further that the circulation is zero when C is any closed path in the fluid at time $t = 0$.

[7]