PART II

Answer **THREE** *questions in this part.*

Each question carries 20% of the total examination marks.

Question 7

(The parts of this question are related.)

(i) Show that $H = p_0/(\rho_0 g)$ has the dimension of length. Here p_0 and ρ_0 are the values of the pressure, p, and the density, ρ, at some datum level $z = z_0$, and g is the magnitude of the acceleration due to gravity. [2]

(ii) Write down the basic equation of fluid statics for a fluid in a gravitational field and the equation of state for a perfect gas. Use these equations to show that both the pressure and the density distributions are exponential functions in z in a static, isothermal, perfect gas in a constant gravitational field. [8]

(iii) Furthermore, show that if both the pressure and the density decrease to a fraction $1/e$ of their respective datum values, p_0 and ρ_0, at $z = z_0$ in a distance H from that datum then $H = p_0/(\rho_0 g)$. (Here e is the base of natural logarithms.) [4]

(iv) Interpret the statement

> $H = p_0/(\rho_0 g)$ is known as the height of the constant density atmosphere

by deriving the pressure distribution for a constant-density model and showing how it relates to H. [3]

(v) Comment, briefly, on the modelling of our Earth's atmosphere and the suitability, or otherwise, of constant density, isothermal, or other models. [3]

Question 8

(i) Consider the time-dependent vector field $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ with Cartesian components

$$u_1 = -y \quad \text{and} \quad u_2 = x - 3t.$$

Show that $\mathbf{u}$ can be used to represent the velocity vector field of an incompressible fluid. Explain why it is not possible to define a velocity potential for this flow.
Determine whether or not the flow is steady. [5]

(ii) Write down the equations describing the stream function for the velocity vector field in Part (i). Hence find the stream function for this flow. Sketch some of the streamlines at time $t = 0$, showing the direction of flow. [7]

(iii) Consider the flow of an inviscid fluid of constant density ρ given by the velocity vector field of Part (i) with body force (per unit mass)

$$\mathbf{F} = 3t\,\mathbf{i} - y\,\mathbf{j}.$$

Find the pressure distribution in the fluid (to within an arbitrary function of time) and hence show that the pressure along a streamline, at $t = 0$, is independent of x. [8]