Question 10

(i) Verify that

$$\phi(r,\theta,z) = \left(Ar + \frac{B}{r}\right)\cos\theta + C\theta$$

could be the scalar potential field for the irrotational flow of an inviscid incompressible fluid where r, θ, z are cylindrical polar coordinates and A, B, C are constants. [5]

(ii) An inviscid incompressible fluid flows irrotationally past an infinite solid cylinder whose surface is $r = a$. At a large distance from the cylinder the velocity of the fluid is $\mathbf{u} = U\mathbf{i}$, where U is a constant. In addition there is a circulation $-\kappa$ around the cylinder. Show that the scalar potential field for this problem is

$$\phi(r,\theta,z) = U\left(r + \frac{a^2}{r}\right)\cos\theta - \frac{\kappa}{2\pi}\theta. \qquad [8]$$

(iii) Ignoring the effects of gravity, obtain an expression for the pressure distribution on the surface of the cylinder. Hence show that the total force per unit length on the cylinder is

$$\mathbf{F} = \rho U\kappa\,\mathbf{j},$$

where ρ is the density of the fluid. [7]