MST322 EXAM 1991

PART I

1. We assume $h = k\rho^{\alpha} S^{\beta} d^{\gamma} \theta^{\delta} g^{\epsilon}$.
Now $[S] = MT^{-2}$, $[h] = L$,
$[\rho] = ML^{-3}$, $[d] = L$, $[\theta] = 1$, $[g] = LT^{-2}$.
Hence $L = (ML^{-3})^{\alpha}(MT^{-2})^{\beta} L^{\gamma} 1^{\delta} (LT^{-2})^{\epsilon}$
$= M^{\alpha+\beta} L^{-3\alpha+\gamma+\epsilon} T^{-2\beta-2\epsilon} 1^{\delta}$.

So
$$\left.\begin{aligned} 0 &= \alpha+\beta, \\ 1 &= -3\alpha+\gamma+\epsilon, \\ 0 &= -2\beta-2\epsilon. \end{aligned}\right\}$$

$\therefore \alpha = -\beta$, $\epsilon = -\beta$, $\gamma = 1-2\beta$.
Hence $h = k\rho^{-\beta} S^{\beta} d^{1-2\beta} \theta^{\delta} g^{-\beta}$
$= k\, d \left(\dfrac{g\rho d^2}{S}\right)^{-\beta} \theta^{\delta}$.

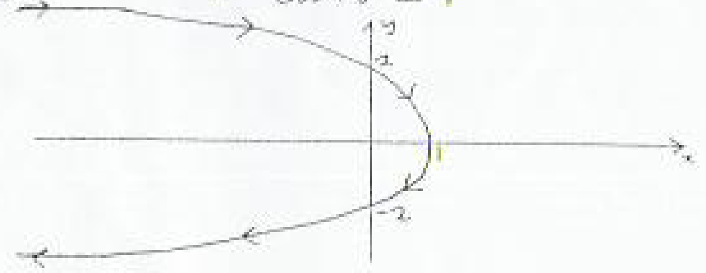
So $h = d\, f\left(\dfrac{g\rho d^2}{S}, \theta\right)$.

2.(i) Stream function $\psi(r,\theta)$ is defined by
$$\left.\begin{aligned} \frac{1}{r}\frac{\partial\psi}{\partial\theta} &= -r^{-1/2}\sin\tfrac{1}{2}\theta, \quad ① \\ \frac{\partial\psi}{\partial r} &= r^{-1/2}\cos\tfrac{1}{2}\theta. \quad ② \end{aligned}\right\}$$

From ②} $\psi(r,\theta) = 2r^{1/2}\cos\tfrac{1}{2}\theta + f(\theta)$.
In ①} $f'(\theta) = 0$.
$\therefore f(\theta) = a$
& $\psi(r,\theta) = 2r^{1/2}\cos\tfrac{1}{2}\theta + a$.

Equations of streamlines are $\psi(r,\theta) = \text{const}$
$\therefore r^{1/2}\cos\tfrac{1}{2}\theta = c$

(ii) If $r=1$ when $\theta=0$, $c=1$.
So $r^{1/2}\cos\tfrac{1}{2}\theta = 1$



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3. As fluid is incompressible, flow rate in two channels is same. So speed in lower channel is
$$u = \frac{3\times 2}{1} = 6\ \text{m s}^{-1}.$$

Let h be difference in height of channel floors.
Choose a streamline in the free surface and apply Bernoulli's equation
$$\frac{p}{\rho} + \tfrac{1}{2}u^2 + gz = \text{const}.$$

Now pressure is constant on free surface.
$\therefore \tfrac{1}{2}\times 3^2 + 10(h+2) = \tfrac{1}{2}\times 6^2 + 10\times 1$
$\therefore h = \frac{27}{20} - 1 = \frac{7}{20}$ (metres)

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4. $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial \xi}\dfrac{\partial \xi}{\partial x} + \dfrac{\partial u}{\partial \phi}\dfrac{\partial \phi}{\partial x} = 2xy\dfrac{\partial u}{\partial \xi} + \dfrac{\partial u}{\partial \phi}$.

$\dfrac{\partial u}{\partial y} = \dfrac{\partial u}{\partial \xi}\dfrac{\partial \xi}{\partial y} + \dfrac{\partial u}{\partial \phi}\dfrac{\partial \phi}{\partial y} = x^2\dfrac{\partial u}{\partial \xi}$