Part I

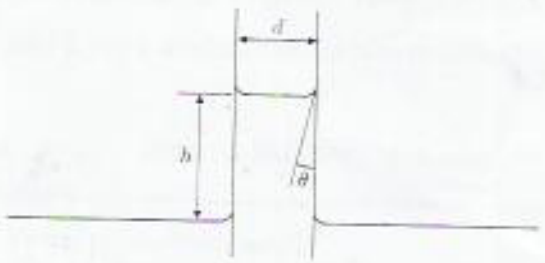
Answer **ALL SIX** *questions in this part.*

The questions in this part are not all worth the same number of marks.

The number of marks assigned to each question is given in square brackets.

Part I as a whole carries 40% of the total examination marks.

Question 1



When a thin tube is dipped into a reservoir of liquid, the free surface of the liquid rises, or falls, inside the tube. This phenomenon is caused by surface tension, defined as force per unit length. Assuming that the height h of the liquid in the tube above the reservoir level depends on the density ρ of the liquid, the surface tension S, the diameter d of the tube, the angle θ of contact between the liquid and the tube and the magnitude g of the acceleration due to gravity, use the method of dimensional analysis to show that

$$h = d\, f\left(\frac{g\rho d^2}{S}, \theta\right),$$

where f is an undetermined function of two variables. [6]

Question 2

Consider the velocity vector field

$$\mathbf{u}(r,\theta,z) = -r^{-\frac{1}{2}} \sin\tfrac{1}{2}\theta\, \mathbf{e}_r - r^{-\frac{1}{2}} \cos\tfrac{1}{2}\theta\, \mathbf{e}_\theta \qquad (r>0, -\pi<\theta<\pi)$$

for the steady two-dimensional flow of an incompressible fluid with respect to cylindrical polar coordinates.

(i) Write down the equations satisfied by the stream function for the velocity field $\mathbf{u}$. Hence find the stream function $\psi(r,\theta)$ for this flow and the equation of the streamlines.

(ii) In particular, find the equation of the streamline which passes through the point $r=1$, $\theta=0$, $z=0$. Hence make a rough sketch of this streamline, indicating the direction of motion. [6]