$$\frac{\partial^2 u}{\partial x^2} = 2y\frac{\partial u}{\partial \xi} + 4x^2y^2\frac{\partial^2 u}{\partial \xi^2} + 4xy\frac{\partial^2 u}{\partial \xi \partial \phi} + \frac{\partial^2 u}{\partial \phi^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = 2x\frac{\partial u}{\partial \xi} + 2x^3y\frac{\partial^2 u}{\partial \xi^2} + x^2\frac{\partial^2 u}{\partial \xi \partial \phi}$$

$$\frac{\partial^2 u}{\partial y^2} = x^4\frac{\partial^2 u}{\partial \xi^2}$$

Substituting in PDE}

$$\left(2x^2y\frac{\partial u}{\partial \xi} + 4x^4y^2\frac{\partial^2 u}{\partial \xi^2} + 4x^3y\frac{\partial^2 u}{\partial \xi \partial \phi} + x^2\frac{\partial^2 u}{\partial \phi^2}\right)$$

$$+\left(-8x^2y\frac{\partial u}{\partial \xi} - 8x^4y^2\frac{\partial^2 u}{\partial \xi^2} - 4x^3y\frac{\partial^2 u}{\partial \xi \partial \phi}\right)$$

$$+ 4x^4y^2\frac{\partial^2 u}{\partial \xi^2} + 6x^2y\frac{\partial u}{\partial \xi} = 0,$$

i.e. $x^2\dfrac{\partial^2 u}{\partial \phi^2} = 0$ or $\dfrac{\partial^2 u}{\partial \phi^2} = 0$

(ii) General solution is

$$u = \phi f(\xi) + g(\xi)$$
$$= x f(x^2y) + g(x^2y)$$

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5 We look for a power series solution

$$y = \sum_{n=0}^{\infty} a_n x^n$$

So $\dfrac{dy}{dx} = \sum_{n=0}^{\infty} n a_n x^{n-1}$ and $\dfrac{d^2y}{dx^2} = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$

From initial conditions $a_0 = 0$ & $a_1 = 1$.

In DE}

$$\sum_{n=0}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=0}^{\infty} n(n-1)a_n x^n$$
$$- \sum_{n=0}^{\infty} 2n a_n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n - \sum_{n=0}^{\infty} (n^2+n+1)a_n x^n = 0$$

So $\quad a_{n+2} = \dfrac{(n^2+n+1)}{(n+1)(n+2)} a_n \qquad n = 0, 1, 2, \ldots$

Now $a_0 = 0$

$\therefore \quad a_2 = a_4 = a_6 = \ldots\ldots = 0$

Also $a_1 = 1$

$\therefore a_3 = \dfrac{3}{2.3} a_1 = \dfrac{1}{2}$

& $a_5 = \dfrac{13}{4.5} a_3 = \dfrac{13}{40}$.

So $\quad y = x + \frac{1}{2}x^3 + \frac{13}{40}x^5 + \ldots\ldots$

Now $p(x) = 1 - x^2$ which has zeros at $x = \pm 1$

So interval of convergence is $-1 < x < 1$

6.(i) $\quad \phi(x, z, t) = f(z)\cos(kx - \omega t)$

Now $\quad \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial z^2} = 0 \qquad 0 < z < h$

$\therefore \quad f''(z) - k^2 f(z) = 0$

So $\quad f(z) = A\cosh kz + B\sinh kz$

Now $\quad \dfrac{\partial \phi}{\partial z} = 0$ at $z = 0$

$\therefore \quad f'(0) = 0 \Rightarrow B = 0$

So $\quad \phi(x, z, t) = A\cosh kz \cos(kx - \omega t)$.

Now $\quad \dfrac{\partial^2 \phi}{\partial t^2} + g\dfrac{\partial \phi}{\partial z} = 0$ at $z = h$

$\therefore \quad -A\omega^2\cosh kh + gAk\sinh kh = 0$

or $\quad \omega^2 = gk\tanh kh$