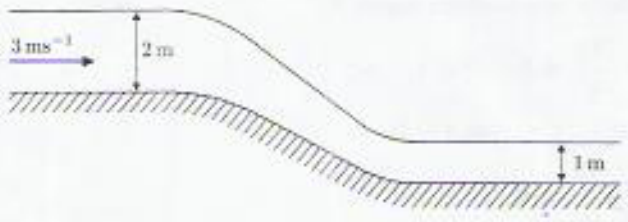
Question 3



Water is flowing in an open rectangular horizontal channel at a depth of 2 metres and a speed of 3 $\mathrm{m\,s^{-1}}$. It then flows smoothly down a chute into another open rectangular horizontal channel of the same width where the depth of water is 1 metre. Assuming the water can be modelled as an inviscid incompressible fluid and that the flow is steady and irrotational, find the speed of the water in the lower channel and the difference in height between the two channel floors. You may take the magnitude g of the acceleration due to gravity as 10 $\mathrm{m\,s^{-2}}$. [5]

Question 4

The function $u(x, y)$ satisfies the partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2} - 4xy\frac{\partial^2 u}{\partial x \partial y} + 4y^2\frac{\partial^2 u}{\partial y^2} + 6y\frac{\partial u}{\partial y} = 0 \qquad (x \neq 0).$$

The characteristic coordinates may be chosen to be

$$\zeta = x^2 y \quad \text{and} \quad \phi = x.$$

(i) Use the coordinates ζ and ϕ to transform the partial differential equation to standard form.

(ii) Hence find the general solution u of the partial differential equation as a function of x and y. [8]

Question 5

Determine the first three non-zero terms of the power series solution about $x = 0$ of the differential equation

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - y = 0$$

which satisfies the initial conditions

$$y(0) = 0 \quad \text{and} \quad \frac{dy}{dx}(0) = 1.$$

State the interval of convergence for the power series solution. [8]