PART II

Answer THREE questions in this part.

Each question carries 20% of the total examination marks.

Question 7

Consider the flow of a viscous fluid of constant density ρ and coefficient of viscosity μ which is bounded by two infinite parallel planes $z = 0$ and $z = h$. The plane $z = 0$ is stationary whereas the plane $z = h$ moves parallel to itself in the direction of the x-axis with velocity $Ue^{-\alpha t}\mathbf{i}$, where U and α are positive constants. There is no applied pressure gradient and the body force due to gravity may be neglected. The fluid flow is modelled by assuming that the velocity field $\mathbf{u}$ is independent of x and y and has no component in the y-direction.

(i) Briefly explain why the above modelling assumptions are reasonable. [2]

(ii) State the boundary conditions on the velocity function $\mathbf{u}$. [2]

(iii) Use the equation of continuity, together with the modelling assumptions and boundary conditions, to show that the velocity field is $\mathbf{u} = u_1\mathbf{i}$ where u_1 is a function of z and t. [4]

(iv) Further, use the Navier-Stokes equation to show that $u_1(z,t)$ satisfies the partial differential equation

$$\frac{\partial u_1}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 u_1}{\partial z^2}.$$ [4]

(v) By assuming a solution of the form

$$u_1(z,t) = e^{-\alpha t}f(z),$$

find the velocity field which satisfies the boundary conditions from part (ii). [6]

(vi) Find the force per unit area on the fixed plane $z = 0$ at time t due to the fluid. [2]