8 ii) e) Model 1 $u_G = 31 \cdot 3\ ms^{-1}$ so 2nd model seems better as u_G is nearer the 'real' one.

9. i) $q(x) = 0$, $\alpha_1 = 0$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 0$ By Theorem 4 (Hbk P29) all eigenvalues are positive (show you have checked it is a Sturm Liouville problem). For $\lambda > 0$ $X = A \cos\sqrt{\lambda}x + B \sin\sqrt{\lambda}x$ – boundary conditions give $B = 0$ and $\sqrt{\lambda}l = \frac{\pi(2n+1)}{2}$ $n = 0, 1, 2, \ldots$ Hence answer.

ii) a) $\frac{\partial u}{\partial x}(0,t) = 0$ $u(l,t) = 0$ $(t \geq 0)$

b) $\cos\frac{\pi x}{2l} \cos\frac{2\pi x}{l} = \frac{1}{2}\cos\left(\frac{3\pi x}{2l}\right) + \frac{1}{2}\cos\left(\frac{5\pi x}{2l}\right)$ (Handbook P11).

c) $u(x,t) = X(x)T(t)$ gives $\begin{cases} X'' + \lambda X = 0 \\ T' + \lambda T = 0 \end{cases}$ $\left(\begin{matrix} X'(0) = X(l) = 0 \\ \text{boundary conditions} \end{matrix}\right)$

so equation in X is same as part (i)

$$X_n(x) = \cos\frac{\pi(2n+1)x}{2l}$$

hence

$$T_n = b_n \exp\left\{-\frac{(2n+1)^2\pi^2 t}{4l^2}\right\}$$

Using principle of superposition.

$$u(x,t) = \sum_{n=0}^{\infty} b_n \exp\left(-\frac{(2n+1)^2\pi^2 t}{4l^2}\right)\cos\left(\frac{\pi(2n+1)x}{2l}\right)$$

Using initial condition $u(x,0) = \sum_{n=0}^{\infty} b_n \cos\left(\frac{\pi(2n+1)x}{2l}\right) = \frac{1}{2}\cos\left(\frac{3\pi x}{l}\right) + \frac{1}{2}\cos\left(\frac{5\pi x}{2l}\right)$

so $b_n = 0$ $n = 0, n \geq 3$ $b_1 = \frac{1}{2}$ $b_2 = \frac{1}{2}$

$$u(x,t) = \frac{1}{2}\exp\left(-\frac{9\pi^2 t}{4l^2}\right)\cos\left(\frac{3\pi x}{2l}\right) + \frac{1}{2}\exp\left(-\frac{25\pi^2 t}{4l^2}\right)\cos\left(\frac{5\pi x}{2l}\right)$$

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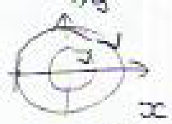
i) $\nabla \cdot \underline{u} = 0$ $\nabla \times \underline{u} = -6\underline{k}$ $\frac{\partial \underline{u}}{\partial t} = 3\underline{j}$ not irrotational, not steady.

ii) $\frac{\partial \psi}{\partial y} = 3y + 3t$ $\frac{\partial \psi}{\partial x} = 3x$, $\psi = \frac{3y^2}{2} + 3ty + \frac{3x^2}{2} + g(t)$ At $t = 0$ $x^2 + y^2 = $ const



iii) textbook P14 Unit 5. iv) Eulers $\rho\left(\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\nabla)\underline{u}\right) = -\nabla p + \rho\underline{F}$

x comp reduces to $\frac{\partial p}{\partial x} = 9\rho x$, y comp to $\frac{\partial p}{\partial y} = 9\rho y$

z comp $\frac{\partial p}{\partial z} = 0$ so $p = p(x,y,t)$ using $P = \frac{9}{2}\rho(x^2 + y^2) + g(t)$.

11 a) Use handbook P33 b) Section 4 i Unit 14. c) Handbook P33 Section 1, 1.

d) use $\phi(x,z,t) = \{D\cosh k(z+h) + E\sinh k(z+h)\}\cos(kx - \omega t)$.

Deep water waves $kh \to \infty$ $e^{-2kh} \to 0$ and $\omega^2 = gk$. so $c^2 = g/k$.

group velocity $c_g = c + k\frac{dc}{dk} = \frac{c}{2}$

ii) a) P31 Unit 8. b) $\frac{\partial \underline{u}}{\partial t} = 0$ (steady) $(\underline{u}\cdot\nabla)\underline{u} = 0$ neglect inertial forces, $\nabla p = 0$ constant pressure $\underline{F} = \underline{0}$ – zero body force so $\nabla^2\underline{u} = \underline{0}$

c) Substitute into LHS of Laplace's equation.

d) Time dependence - waves vary with time whereas only steady flow is considered in Part (ii).