1983

Part I

Answer **ALL SIX** *questions in this part.*

The questions in this part are not all worth the same number of marks.

The number of marks assigned to each question is given in square brackets.

Part I as a whole carries 40% of the total examination marks.

Question 1

Assuming that the magnitude of the drag force F on a ship is a function of the coefficient of viscosity μ and the density ρ of the fluid, the speed v of the ship, the acceleration due to gravity g and the size of the ship (which depends on its length ℓ), show that

$$F = \rho v^2 \ell^2 f\left(\frac{\mu}{\ell \rho v}, \frac{\ell g}{v^2}\right),$$

where f is an arbitrary function. [6]

(Hint: Express the other powers in terms of the powers of μ and g.)

Question 2

The function $u(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0.$$

In the plane, characteristic coordinates may be chosen to be

$$\zeta = y - x \quad \text{and} \quad \phi = y + x.$$

(i) Use the coordinates ζ and ϕ to transform the partial differential equation to standard form.

(ii) Hence find the general solution u as a function of x and y. [7]

Question 3

Determine the first three non-zero terms of the power series solution about $x = 0$ of the differential equation

$$(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

such that $y(0) = 1$ and $\frac{dy}{dx}(0) = 0$. [7]

Question 4

The vector field

$$\mathbf{u} = r\sin\theta\,\mathbf{e}_r + 2r(1 + \cos\theta)\mathbf{e}_\theta$$

represents the velocity field of an incompressible steady two-dimensional flow, referred to cylindrical polar co-ordinates.

(i) Write down the equations describing the stream function for the velocity vector $\mathbf{u}$. Hence find the stream function for this flow and the equation of the streamlines.

(ii) Find the equation of the streamline which passes through the point given by $r = 1$, $\theta = \pi$. Sketch this streamline. Describe the flow along this streamline. [6]