Question 5

Let r be rotation anticlockwise through $\pi/2$; then the direct symmetry group is

$G = \{e, r, r^2, r^3\}$. 1 $-\frac{1}{2}$ each error

Under e, there is no restriction on the colouring.
Under r, all sides must be the same colour.
Under r^2, opposite sides must be the same colour.
Under r^3, all sides must be the same colour.
So

$|\mathrm{Fix}(e)| = 3^4 = 81, \quad |\mathrm{Fix}(r)| = 3, \quad |\mathrm{Fix}(r^2)| = 3^2 = 9, \quad |\mathrm{Fix}(r^3)| = 3.$ 3

By the Counting Lemma, the number of equivalence classes is

$\frac{1}{4}(81 + 3 + 9 + 3) = 24.$ 1

Question 6

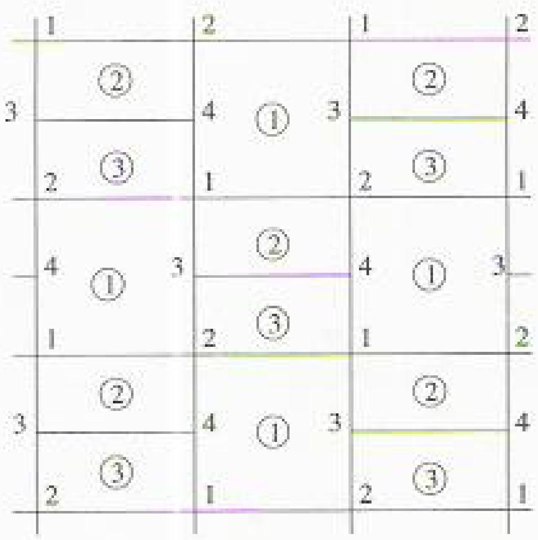
(a) The order of G is 25, so the possible orders are 1, 5 and 25. 1 $-\frac{1}{2}$ each error or omission

(b) The identity is unique, as is the whole group; i.e. just one subgroup of order 1, one of order 25. 2 1M, 1A

G contains 24 elements of order 5 and each subgroup of order 5 contains four of them (and the identity); any two such sets of four elements of order 5 are disjoint. 1
So, there are six subgroups of order 5. 1

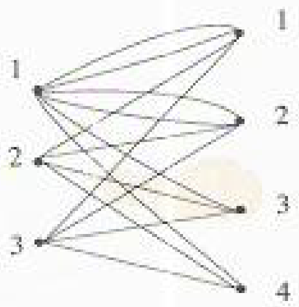
Question 7

(a)



3

(b)



tile orbits vertex orbits 2