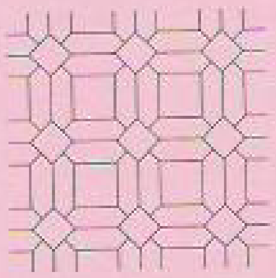
Part I

You may attempt **ALL** *the questions in this part and are advised to spend about* **90 minutes** *on it.*

The marks for each question are given beside the question.

Question 1

This question concerns the following tiling, $\mathcal{T}$.



$\mathcal{T}$

(a) Write down all the tile types and vertex types in $\mathcal{T}$. [3]

(b) The tiling $\mathcal{T}$ has three different (non-congruent) tile shapes. Can this number be increased by applying an affine transformation to $\mathcal{T}$? If so, what is the maximum number of tile shapes that can be so produced; if not, why not? [2]

Question 2

The dihedral group D_4 is given by the presentation

$$D_4 = \langle r, s : r^4 = e,\ s^2 = e,\ sr = r^3 s \rangle$$

and the elements are written in standard form as

$$r^m \text{ and } r^m s, \quad m = 0, \ldots, 3.$$

A group action of D_4 on D_4 is defined by

$$g \wedge x = gxg^{-1}.$$

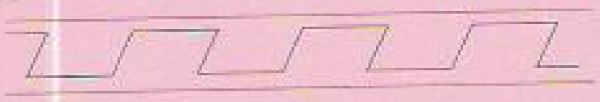
(You may assume that this *is* a group action.)

(a) Express $s \wedge r$ in standard form. [2]

(b) Find $\mathrm{Orb}(r)$. [3]

Question 3

Use the algorithm of *Unit IB3* to determine the type of the following frieze.



Your answer should make clear which questions you ask and what the answers are. [5]

Question 4

Let ϕ and ψ be the functions from $\mathbb{Z}_{10}$ to itself defined by

$$\phi : k \mapsto 2k \quad \text{and} \quad \psi : k \mapsto 3k,$$

which you may *assume* are homomorphisms.
One of these is an *isomorphism* from $\mathbb{Z}_{10}$ to itself, the other is not. Explain which is which, giving brief reasons for your answer. [5]