Question 14

(a) There are eight rotations through $\pm 2\pi/3$ about axes through opposite corners, and three rotations through π about axes through mid-points of opposite faces, together with the identity. 4

We think that the $\pm 2\pi/3$ rotations are easier to see for $\mathcal{C}_2$ and the others for $\mathcal{C}_1$, so the given information about the group G^+ is not totally redundant.

(b) The identity gives four 1-cycles.
Each of the rotations about an axis through opposite corners gives a 3-cycle.
Each of the rotations about an axis through mid-points of opposite faces gives a pair of 2-cycles.
Thus G^+ contains 12 *even* permutations and so

$$G \cong A_4.$$ 4

(c) G_1 contains central inversion, so

$$G_1 \cong A_4 \times C_2.$$ 3

G_2 contains, for example, reflection in a plane containing a pair of opposite edges, say the one that also contains the diagonals labelled 2 and 4. This corresponds to the permutation (13).
Hence, G_2 is a group of permutations of four objects of which A_4 is a *proper* subgroup. The only possibility is

$$G_2 \cong S_4.$$ 4