Question 8

This may be tackled by looking at either the primary decomposition or the canonical decomposition.

Primary

$$A_1 \cong (\mathbb{Z}_2 \times \mathbb{Z}_3) \times \mathbb{Z}_4 \times (\mathbb{Z}_3 \times \mathbb{Z}_5) \cong (\mathbb{Z}_2 \times \mathbb{Z}_4) \times (\mathbb{Z}_3 \times \mathbb{Z}_3) \times \mathbb{Z}_5$$

$$A_2 \cong (\mathbb{Z}_2 \times \mathbb{Z}_9) \times (\mathbb{Z}_4 \times \mathbb{Z}_5) \cong (\mathbb{Z}_2 \times \mathbb{Z}_4) \times \mathbb{Z}_9 \times \mathbb{Z}_5$$ 4 2 each decomposition

The 3-primary component of A_1 is $\mathbb{Z}_3 \times \mathbb{Z}_3$, that of A_2 is $\mathbb{Z}_9$. These are different, so the groups are *not* isomorphic. 1

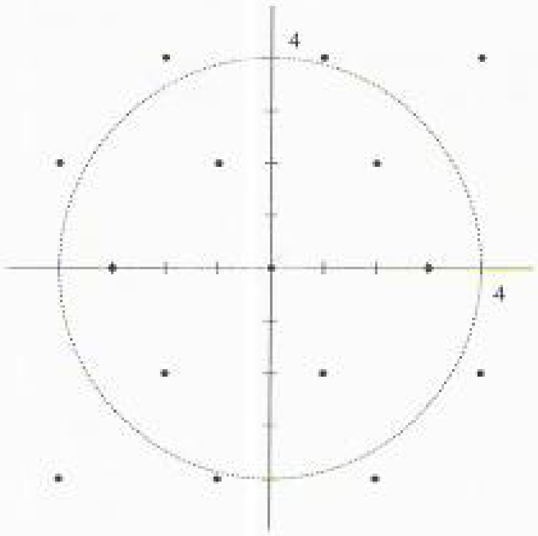
Canonical

$$A_1 \cong \mathbb{Z}_6 \times \mathbb{Z}_{60}$$

$$A_2 \cong \mathbb{Z}_2 \times \mathbb{Z}_{180}$$ 4 2 each decomposition

These are different canonical decompositions, so the groups are *not* isomorphic. 1

Question 9



(a) $(0,0), (3,0), (-3,0), (2,2), (-2,-2), (1,-2), (-1,2)$ 3 $-\frac{1}{2}$ each error or extra point

(b) $\{(1,-2), (2,2)\}$ 2 $-\frac{1}{2}$ each error.

$[(1,-2)$ could be replaced by $(-1,2)$ and $(2,2)$ could be replaced by $(-2,-2)$.]

Question 10

As G has order 75 and $Z(G)$ is a subgroup, the *possible* orders for $Z(G)$ are 1, 3, 5, 15, 25 and 75. 1

$|Z(G)| = 75$ means G is Abelian, contradicting what is given. 1

$|Z(G)| = 25$ means that the quotient $G/Z(G)$ has order 3, and hence is cyclic, and so G is Abelian (*Unit GR4*), a contradiction.

$|Z(G)| = 15$ means that the quotient $G/Z(G)$ has order 5, and hence is cyclic, and so G is Abelian (*Unit GR4*), a contradiction. 2 1M, $\frac{1}{2}$each case

Thus $Z(G)$ has order 1, 3 or 5, i.e. at most 5. 1