Solutions to Part I

Question 1

(a) Tile types: $[3,3,3,3]$, $[3,3,3,3,3,3]$, $[3,3,3,3,3,3,3,3]$. $1\frac{1}{2}$ $-\frac{1}{2}$ each error
Vertex types: $(4,6,6)$, $(6,6,8)$. $1\frac{1}{2}$ $-\frac{1}{2}$ each error Deduction is down to zero; we do not give negative marks!

(b) The number can increase to 4. This is because not all the hexagonal tiles map to each other by a translation or by a translation composed with rotation through π. 2

Question 2

(a) $s \wedge r = srs^{-1} = (r^3 s)s^{-1} = r^3$. 2

(b) We have

$$\begin{aligned} r^m \wedge r &= r^m r r^{-m} = r, \\ r^m s \wedge r &= r^m srs^{-1} r^{-m} \\ &= r^m r^3 r^{-m} \quad \text{from part (a)} \\ &= r^3. \end{aligned}$$

This covers the action of all the group elements, so

$$\mathrm{Orb}(r) = \{r, r^3\}.$$

3 2M, 1A

Question 3

Using the diagram on p. 13 of the *Handbook*, the questions and answers are:

v? no;
h? no;
g? no;
r? yes.

Thus the frieze is of Type 5 (or f_r or $p112$). [Any one of the three forms is acceptable.]

5 1 for each line of algorithm, 1 for conclusion.

Question 4

The function ϕ is *not* an isomorphism because $\frac{1}{2}$

$$\phi(0) = \phi(5) = 0,$$

so ϕ is not one-one.

The function ψ is an isomorphism:

One-one

$$\psi(k) = \psi(l) \Rightarrow 3k = 3l \Rightarrow k = l$$

(as 3 is not a factor of 10), so that ψ is one-one. 2

Onto

Because $\mathbb{Z}_{10}$ is finite, one-one implies onto. 1