MT365/W

Third-level Course Examination 1996
Graphs, Networks and Design

Friday 25th October 1996 10.00 am – 1.00 pm

Time allowed: 3 hours

There are TWO parts to this paper.

52% of the available marks are assigned to Part 1 (4 marks per question) and 48% are assigned to Part 2 (12 marks per question). You should not expect to be awarded a distinction unless you obtain high marks on both Part 1 and Part 2.

In Part 1 you should attempt as many questions as you can. Please begin each new question on a new page, and indicate clearly the number of the question you are attempting.

In Part 2 you should attempt not more than FOUR questions, including at least one question from each section. Please begin each new question on a new page, and *write the numbers of the Part 2 questions you attempt on the front page of the answer book for Part 2.*

Write your answers to Parts 1 and 2 in separate answer books. Additional answer books are available from the invigilator, if needed.

At the end of the examination

Attach together, using the paper fastener provided, the answer books in which you have answered questions from Part 1 and Part 2.

Check that you have written your name, personal identifier and examination number on each answer book used. **Failure to do so will mean that your work cannot be identified.**

YOU MUST NOT USE A CALCULATOR IN THIS EXAMINATION.

Part 1

Part 1 carries 52% of the total marks for the examination (4 marks per question). Answer as many questions as you can from this part. It will help the examiners if you answer the questions in the order in which they are set.

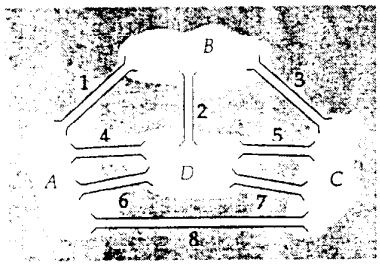
Write your answers in one of the answer books provided.

DO NOT use the same answer book for Part 1 as for Part 2.

Please begin each Part 1 question on a new page.

Question 1

The land of Combinatoria consists of four islands linked by eight bridges, as follows:



(a) Draw a graph that represents this arrangement of islands and bridges.

(b) The citizens of Combinatoria wish to go for a walk that crosses each bridge exactly once.

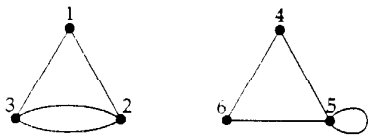
(1) Is this possible, if the walk must start and finish at the same island?

(2) Is this possible, if the walk need not start and finish at the same island?

(In each case, write down the order of the bridges traversed in such a walk, or give a reason why a walk is not possible.)

Question 2

Consider the following graph G.



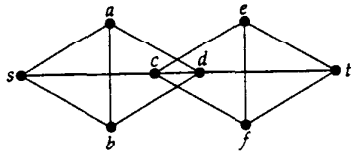
(a) Write down the adjacency matrix of G.

(b) Verify that the handshaking lemma holds for G.

(c) Draw a simple connected graph with the same degree sequence as G.

Question 3

Consider the following graph G.



(a) Write down the values of $\kappa(G)$ and $\lambda(G)$, and justify each answer by stating an appropriate set of vertices/edges whose removal disconnects G.

(b) Write down (without explanation)

(1) two vertex-disjoint st-paths;

(2) three edge-disjoint st-paths.

Question 4

Let P, Q, R, S, T, U be six points in the plane.

(a) Explain briefly why each of $\langle P, Q, R\rangle$ and $\langle S, T, U\rangle$ must be contained in $\langle P, Q, R, S, T, U\rangle$.

(b) Draw an example in which each point of $\langle P, Q, R, S, T, U\rangle$ lies in at least one of $\langle P, Q, R\rangle$ and $\langle S, T, U\rangle$.

Question 5

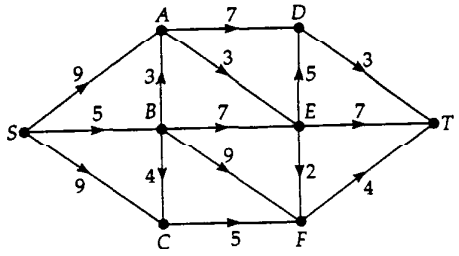
The following table gives the distances between five cities A, B, C, D, E.

	A	B	C	D	E
A	–	5	7	3	7
B	5	–	4	4	6
C	7	4	–	5	7
D	3	4	5	–	6
E	7	6	7	6	–

Use Kruskal's algorithm to find a minimum connector for these cities, and state its total length. (Your answer should state clearly the order in which the edges are chosen, and why).

Question 6

Consider the following weighted digraph D.



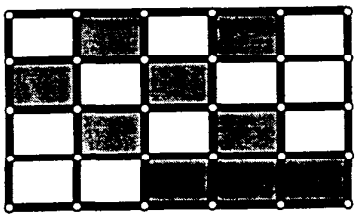
The shortest path algorithm is applied to the digraph D to find a shortest path from S to T.

(a) Write down the potentials assigned to the vertices in the order in which they are assigned. (No explanation is required.)

(b) Write down a shortest path from S to T.

Question 7

(a) Determine whether the following braced rectangular framework is rigid, by constructing a suitable bipartite graph.

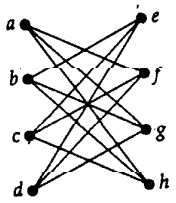


(b) If the above braced rectangular framework is rigid, determine whether the bracing is a minimum bracing.

If it is not rigid, write down an extra brace whose addition would make the framework rigid.

Question 8

Consider the following planar graph G.



(a) Construct a plane drawing of G, and write down the number of faces.

(b) What is the chromatic number of G? (Give a reason for your answer.)

Question 9

Four applicants a, b, c, d are rated for their ability to carry out four tasks w, x, y, z, according to the following cost matrix (with lower cost implying greater ability).

	w	x	y	z
a	7	3	4	8
b	9	5	3	10
c	11	4	4	6
d	5	6	11	7

Construct the first revised cost matrix and the first partial graph used in applying the Hungarian algorithm to find the optimum assignment of applicants to jobs.
(Do **not** proceed with the algorithm.)

Question 10

Consider the following code C:

{0000000000, 0011101010, 1100010110, 1111111111}.

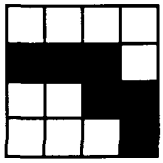
(a) How many errors are detected and corrected by the code C? (Justify your answer.)

(b) A codeword is transmitted and the binary word 1111101010 is received. Determine the codeword that is most likely to have been transmitted.

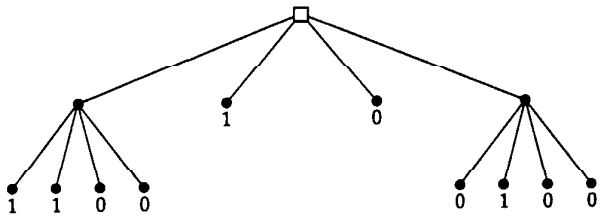
(c) Is C a linear code? (Justify your answer.)

Question 11

(a) Construct the pruned quad tree representation of the following 4×4 screen image.

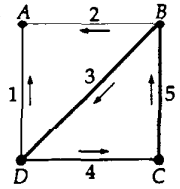


(b) Draw the 4×4 screen image corresponding to the following pruned quad tree.



Question 12

The following oriented graph represents an electrical network with five components.



Using the spanning tree with edges BA, BD, CB, write down:

(a) the fundamental cycles and the corresponding voltage equations;

(b) any **two** fundamental cutsets and the corresponding current equations.

Question 13

Consider the following incomplete block design Δ.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	*B*	*A*	*B*	*A*	*D*	*A*	*A*	*B*	*C*
B	*C*	*C*	*D*	*C*	*E*	*D*	*B*	*D*	*D*
E	*F*	*E*	*F*	*F*	*F*	*E*	*C*	*E*	*F*

(a) Is Δ a resolvable design?
If so, write down the replicates; if not, explain why not.

(b) Is Δ a balanced design?
If so, write down the value of λ; if not, explain why not.

Part 2

Part 2 is divided into three sections. You should attempt **not more than FOUR** *questions from this part, including* **at least one from each section.**

Each question in this part is allotted **12 marks.**

Show all your working.

To help the examiners, please write the numbers of the questions you have attempted in Part 2 at the foot of the front cover of your answer book for this part.

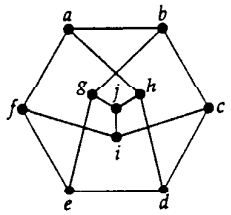
DO NOT use the same answer book as you used for Part 1.

Please begin each Part 2 question on a new page.

Section A Graphs

Question 14

Consider the following graph G.



(a) Prove that

(1) G has no Hamiltonian cycle that includes the edges jh, ha and ab;

(2) G has no Hamiltonian cycle that includes the edges jh, ha and af. (8 *marks*)

(b) Use the results of part (a) to prove that G has no Hamiltonian cycle. (4 *marks*)

Question 15

(a) Draw the labelled tree that corresponds to the Prüfer sequence **(1, 1, 1)**. (2 *marks*)

(b) Draw all the unlabelled trees with five vertices. (2 *marks*)

(c) Write down the total number of labelled trees with five vertices. (1 *mark*)

(d) Let T be the unlabelled tree that can be labelled to give the labelled tree of part (a). Which Prüfer sequences other than **(1, 1, 1)** correspond to labelled trees that are also labellings of T? (2 *marks*)

(e) Prove that there are exactly two non-isomorphic unlabelled trees with five vertices each of which can be labelled to give exactly 60 different labelled trees. (5 *marks*)

Question 16

Four feature items A, B, C, D are available for a broadcast slot on a television magazine programme. The broadcast slot has a maximum length of ten minutes. Each feature item is given a value rating. The value rating and length (in minutes) of each item are shown in the following table.

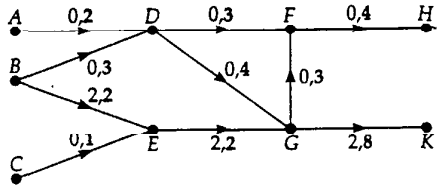
item	A	B	C	D
length	2	5	8	3
value	3	10	14	5

Use the branch-and-bound method to determine which items should be used in the broadcast slot, so as to maximize the total value rating. (Show all your working.) (12 *marks*)

Section B Networks

Question 17

Consider the following network in which the numbers next to each arc are the value of the flow and the capacity of the arc.



(a) Convert the above network to a basic network. (2 *marks*)

(b) Use the maximum flow algorithm to find a maximum flow from the vertices A, B, C to the vertices H, K, and a corresponding minimum cut.

(Show all your working. It is not necessary to consider the vertices in alphabetical order.) (5 *marks*)

(c) A single arc is to be added to the network so as to increase the value of the maximum flow from the vertices A, B, C to the vertices H, K. The possible additional arcs are

AE (capacity 6), EF (capacity 4), CD (capacity 5).

Determine which of these arcs gives the largest value for the maximum flow, and write down this value. (5 *marks*)

Question 18

The durations (in days) of the activities of a project are:

activity	A	B	C	D	E	F	G	H
duration	5	6	8	5	7	2	4	3

The precedence relations are:

B must follow A and C
D must follow C and E
F must follow E
G must follow B and D
H must follow D and F

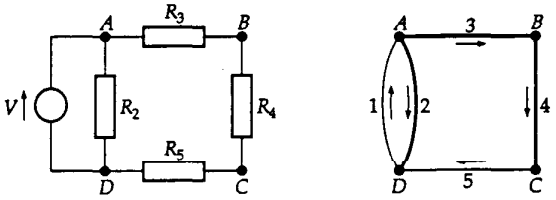
(a) Use the algorithm given in *Networks* 2 to number the vertices and to draw a fully-labelled activity network in which the activities are represented by vertices. (5 *marks*)

(b) Find a critical path by inspection, or otherwise. Write down the length of the critical path and the latest and earliest starting times for each activity of the project. (4 *marks*)

(c) Because of a shortage of workers, the completion of one of the activities B, D or F is delayed by 4 days. By first calculating the float for each of these three activities, calculate the delay in the completion of the whole project in each case. (3 *marks*)

Question 19

Consider the following electrical network and oriented graph.



(a) Write down the incidence matrix of the oriented graph above. (2 *marks*)

(b) Taking vertex D as the reference vertex, write down the reduced incidence matrix partitioned into two parts, the first corresponding to the branches and the second to the chords. (Use the spanning tree shown above with thick edges.) (2 *marks*)

(c) Obtain the fundamental cycle matrix from the reduced incidence matrix.

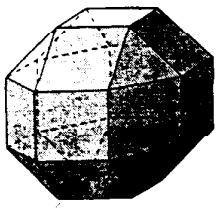
(*Hint*: the inverse of the tree part of the reduced incidence matrix is obtained by tracing the path from each vertex of the tree to the reference vertex. In the inverse matrix each column corresponds to the path from a vertex to the reference vertex: an element is 1 if the corresponding branch is traversed in the same direction as the reference direction, –1 if traversed in the opposite direction, and 0 otherwise.) (6 *marks*)

(d) Apart from the fundamental cycle and cutset equations, which additional equations are needed in order to find all the currents and voltages in the above circuit? Write down these additional equations. (2 *marks*)

Section C Design

Question 20

The following diagram illustrates a *small rhombicuboctahedron*.



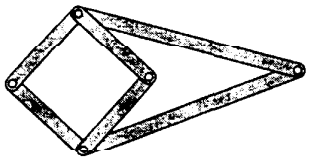
(a) Write down the numbers of vertices, edges, triangular faces and square faces of a small rhombicuboctahedron. (3 *marks*)

(b) Sketch a plane drawing of the graph of the small rhombicuboctahedron, and verify that Euler's formula holds for your drawing. (6 *marks*)

(c) Explain why all the faces of the dual of a small rhombicuboctahedron are quadrilaterals. Write down the degrees of the vertices of the dual, and the number of vertices of each degree. (3 *marks*)

Question 21

Consider the following planar kinematic system in which all the joints are revolute joints.



(a) State the numbers of links and joints of each type in the system. (3 *marks*)

(b) Write down an appropriate mobility formula for this system, and hence derive its mobility. (2 *marks*)

(c) Write down the number of possible systems that can be obtained from the above system by expanding each ternary joint into binary joints. (1 *mark*)

(d) Of the systems in part (c), it is possible to find two with non-isomorphic interchange graphs. Sketch two such systems and their interchange graphs. (6 *marks*)

Question 22

(a) Write down the cyclic block design Δ with 7 varieties and first block $\{0, 1, 3\}$, and its complement $\overline{\Delta}$. (3 *marks*)

(b) Let C be the linear code whose codewords are 0000000, 1111111 and the rows of the incidence matrices of Δ and $\overline{\Delta}$. Write down all the codewords of C. (3 *marks*)

(c) Write down the generator matrix for C in standard form, and hence find a parity check matrix for C. (4 *marks*)

(d) A codeword is transmitted and the binary word 1101100 is received. Determine the codeword of C that is most likely to have been transmitted. (2 *marks*)

[END OF QUESTION PAPER]