Question 4

(a) We begin by observing that S_1 and S_2 satisfy (11.8) of Falconer with $c_1 = c_2 = 3/4$.

The conditions given in the question ensure that $S_1(p_1)$, $S_2(p_1)$ and p_2 are not all collinear and so it follows from Example 11.4 of Falconer that, if conditions (11.9) and (11.10) of Falconer are satisfied, then F is a self-affine curve with box dimension

$$1 + \frac{\log(c_1 + c_2)}{\log 2} = 1 + \frac{\log(3/2)}{\log 2} = 1.58$$

to two decimal places.

So, we must find values of a_i and b_i $(i = 1, 2)$ such that F passes through the points given in the question and such that equations (11.9) and (11.10) of Falconer are satisfied.

We begin by noting that (11.9) is satisfied since

$$1/2 = 1/m < c_i = 3/4 < 1, \text{ for } i = 1, 2.$$

For (11.10) to be satisfied and the curve F to pass through the required points, we must have

$$S_1(p_2) = S_2(p_1) = (1/2, 1),$$

where

$$p_1 = \left(0, \frac{b_1}{1 - c_1}\right) = (0, 4b_1) = (0, 0),$$

so that $b_1 = 0$, and

$$p_2 = \left(1, \frac{a_2 + b_2}{1 - c_2}\right) = (1, 4(a_2 + b_2)) = (1, 0),$$

so that $a_2 + b_2 = 0$. Now

$$S_1(1, 0) = (1/2, a_1 + b_1) = (1/2, a_1) \text{ and } S_2(0, 0) = (1/2, b_2).$$

So we must have

$$(1/2, a_1) = (1/2, b_2) = (1/2, 1)$$

Thus

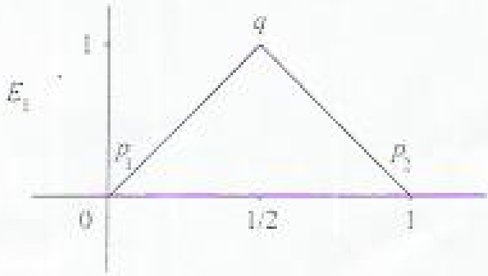
$$a_1 = b_2 = 1 \text{ and hence } a_2 = -1.$$

So, to summarize,

$$a_1 = 1, \quad b_1 = 0$$
$$a_2 = -1, \quad b_2 = 1.$$

13 marks

(b) Putting $q = S_1(p_2) = S_2(p_1) = (1/2, 1)$, we see that the curve E_1 is made up of the straight line segments
$[S_1(p_1), S_1(p_2)] = [p_1, q] = [(0, 0), (1/2, 1)]$, and $[S_2(p_1), S_2(p_2)] = [q, p_2] = [(1/2, 1), (1, 0)]$
as shown below.



The curve E_2 is made up of the straight line segments $[S_1(p_1), S_1(q)]$, $[S_1(q), S_1(p_2)]$, $[S_2(p_1), S_2(q)]$ and $[S_2(q), S_2(p_2)]$.

Now

$$S_1(q) = S_1(1/2, 1) = (1/4, 1/2 + 3/4) = (1/4, 5/4)$$