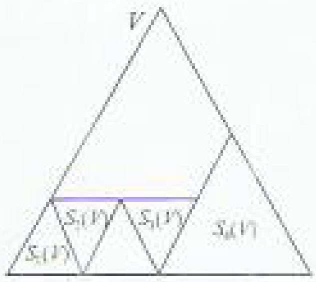
(b) F is a non-empty compact set since it is generated by the polygonal curve E_1.

The set F is invariant under similarities S_1, S_2, S_3 and S_4 which have ratios $c_1 = c_2 = c_3 = 1/4$ and $c_4 = 1/2$.

The following picture shows that the open set condition holds.



It follows from Theorem 9.3 of Falconer that $\dim_H F = \dim_B F = s$, where s is given by

$$1 = \sum_{i=1}^{4} c_i^s = 3(1/4)^s + (1/2)^s.$$

Putting $x = (1/2)^s$, we have

$$3x^2 + x - 1 = 0$$

and so

$$x = \frac{-1 \pm \sqrt{13}}{6}.$$

Since $x = (1/2)^s > 0$, it follows that $(1/2)^s = (-1 + \sqrt{13})/2$ and so $-s \log 2 = \log((-1 + \sqrt{13})/2)$. Thus

$$s = \frac{-\log((-1 + \sqrt{13})/2)}{\log 2}.$$

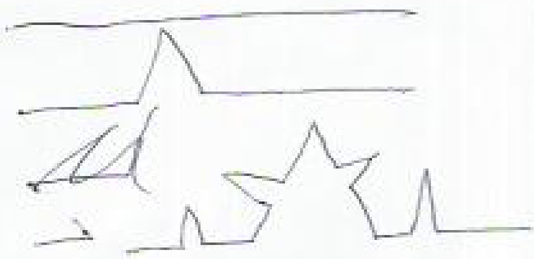
13 marks

(c) There are several possibilities here.

A curve with four straight line segments is shown below.

The curve generated will be invariant under three similarities which have ratios of $1/4$ and one which has ratio $1/2$ and so, by Theorem 9.3 of Falconer, will have the same dimension as F.

A curve with seven straight line segments is shown below.



Here one of the four segments of E_1 has been replaced by the appropriate part of the curve E_2 and so the limiting curve will be equal to F.

6 marks

[25 marks]