

Question 1

Let F consist of those real numbers in $[0, 1]$ whose non-terminating decimal expansions contain only the digits 2, 5 and 8. Thus, if $E_0 = [0, 1]$, then $F = \bigcap_{k=0}^{\infty} E_k$, where E_{k+1} is constructed by removing all but the second, fifth and eighth tenths of each interval in E_k .

- Sketch the sets E_0 , E_1 and E_2 . 3
- Show that F is non-empty and compact. 3
- Use Proposition 4.4 and Example 4.6 of Falconer to determine the Hausdorff and box dimensions of F . 11
- Let $f_k: [0, 1] \rightarrow \mathbb{R}$ be the function defined by $f_k(x) = x + kx^2$, for $k = 0, 1, 2, \dots$. Show that f_k is a Lipschitz function, for $k = 0, 1, 2, \dots$, and hence determine $\lim_{k \rightarrow \infty} \bigcup_{k=0}^{\infty} f_k(F)$. 8

Question 2

Let $F = \bigcap_{k=0}^{\infty} E_k$, where E_0 is the square $\{(x, y) : 0 \leq x, y \leq 1\} \subset \mathbb{R}^2$ and E_{k+1} is constructed by dividing up each square in E_k with 2 equally spaced horizontal lines and 2 equally spaced vertical lines (thus giving 9 equal subsquares) and then removing all but the four corner subsquares.

- Sketch the sets E_0 , E_1 and E_2 . 3
- Use convenient parts of Equivariance definition 3.1 of Falconer to determine the box dimension of F . 4
- Use results from Chapter 4 of Falconer's *Fractal Geometry* with the result of part (b) to determine the Hausdorff dimension of F . 8
- Show that F is totally disconnected. 6

PART B You should attempt TWO questions in this part.

Question 3

Let F be the curve generated by the curve E_1 shown below where the second quarter of the interval $[0, 1]$ has been replaced by the other two sides of the equilateral triangle based on the removed segment.



- Sketch the next two stages, E_2 and E_3 , in the construction of F . 6
- Using results from Chapter 9 of Falconer, determine the Hausdorff and box dimensions of F . 11
- Draw two distinct curves (not equal to E_k for any $k \in \mathbb{N}$) each of which generates a curve of the same Hausdorff dimension as F . One of the curves should have four straight line segments and the other should have seven straight line segments. Justify your answer briefly. 10