<u>MS221 – 2000 Solutions</u>
*****_qns 1,3,4 no longer in sy;llabus**

<u>Qn.1</u> 5764/9999 = 524/909

<u>Qn.2</u> (a) discriminant=$0^2-4.1.4<0$
(n.b. discriminant is no longer in syllabus)
(b) (i) solve $x^2-4x-12=0$
=> (x-6)(x+2)=0
=> x=6 or x=-2
so points are (6,0) and (-2,0).
(ii) $4(-y)^2= 4y^2$
(c) C is the only one consistent with results (b).

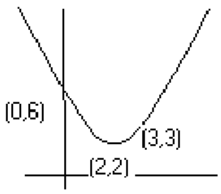
<u>Qn.3</u>(a) (i) $trans_{-6}$ (ii) ref_4
(b) 6-(2p-x)=x+10 => p=-2

<u>Qn.4</u> (i) **b-a** (ii) ¾(**b-a**)
(b) **a** + ¾(**b-a**) = ¾**b** + ¼**a**

(c) $\begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$

<u>Qn.5</u> (a) no
(b) f(½) = f(-¾) = -½

<u>Qn.6</u> (a) $(1+k)^7=$
$1+7k+21k^2+35k^3+35k^4+21k^5+7k^6+k^7$
(b) (i) Put k=-0.005
first 3 terms=1–0.035+0.000525
= 0.965525 = 0.966 to 3 d.p.
(ii) $35k^3$= 0.0000044, so this and subsequent



terms are small.

<u>Qn.7</u> (a) solve $x^2-4x+6=x$
=> $x^2-5x+6=0$ => (x-3)(x-2)=0
=> fixpoints are 3 and 2
(b) see above
(c) Parabola is flat at (2,2) so fixpoint at 2 is (super) attracting. Parabola is steeper than y=x at (3,3) so fixpoint at 3 is repelling.

<u>Qn.8</u> (a) $\sin\theta = 4/\sqrt{65}$ $\cos\theta = 7/\sqrt{65}$

(b) $\frac{1}{\sqrt{65}}\begin{pmatrix} 7 & -4 \\ 4 & 7 \end{pmatrix}$

(c) Inverse = $\frac{1}{\sqrt{65}}\begin{pmatrix} 7 & 4 \\ -4 & 7 \end{pmatrix}$ which represents the rotation which maps 7y=4x onto the x-axis.

<u>Qn.9</u> (a) $\frac{x}{\sqrt{1-x^2}} + \arcsin x$
using product rule

(b) $-\frac{1}{t}\sin(\ln t)$ by chain rule

<u>Qn.10</u> (a)
I=$-(x+2)\exp(-x)+\int\exp(-x)dx$
$=-(x+2)\exp(-x)-\exp(-x)+c$
$=-(x+3)\exp(-x)+c$
(b) du=$\sec^2$ x dx
I= $\int\frac{du}{u} = \ln u + c = \ln(1+\tan x)+c$

<u>Qn.11</u> (a) $f(x)=1-3x+9x^2-27x^3$
(b) (-1/3 < x < 1/3)
(c) $-3+18x-81x^2$

<u>Qn.12</u> (a) $\frac{dy}{y^{\frac{1}{2}}} = \cos x dx$
$2y^{1/2} = \sin x + const.$
$y = (\frac{1}{2}\sin x + c)^2$
(b) y=1 when x=0 => c=1
so particular soln. is
$y = (\frac{1}{2}\sin x + 1)^2$

<u>Qn.13</u> (a) $<4, \frac{\pi}{6}>, <4, \frac{7\pi}{6}>$
(b) $2\sqrt{3}+2i$, $-(2\sqrt{3}+2i)$

<u>Qn.14</u> (a) 6^2=36=2(mod 17) so remainder is 2.
(b) In mod 17 arithmetic 6 is congruent to 40.
$6^2=2$, $6^3=12$, $6^4=4$ so $6^7=14$ (48-34) and therefore remainder is 14.

<u>Qn.15</u> (a) b (b)(i) d (ii) c
(c) the addition table for Z_4, since group is Abelian & there are two self-inverse elements.
(Match a=1, b=0, c=2, d=3)

<u>Qn.16</u> (a) $a(n) \wedge c(n)$
(b) 30
(c) (i) $d(n) \Rightarrow (a(n) \wedge b(n))$
(ii) if an integer is divisible by 90, it is divisible by both 5 and 6.

Qn.17 (a) (i) a reflection about such a line results in a complete overlap of the curve and its transformation.
(ii) A. one - Oy B. none
C. two : y=-2x and y= ½x
D. two : major and minor axes.
(b) B,C and D: centre 0,angle ©
(c) (i) The remaining four isometries are:
e : identity
$q_{\pi/3}, q_{2\pi/3}$: reflections about OB and OC
$r_{4© /3}$: rotation of 4©/3 clockwise about the origin.

(ii) $\begin{matrix} e & q_{2\pi/3} \\ q_{\pi/3} & r_{4\pi/3} \end{matrix}$

Qn.18 (a) For eigenvalues solve

$$(4-k)(-1-k)+6=0$$

$$-4-3k+k^2+6=0$$

$$k^2-3k+2=0 \Rightarrow k=1, k=2$$

Eigenlines are given by

$$\begin{pmatrix} 3 & -6 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0, \begin{pmatrix} 2 & -6 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0$$

that is x-2y=0, x-3y=0

(b)E-vectors are $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ so one Q is

$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$, Q^{-1} is $\begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$.

(c) $M^{10}=\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^{10}\begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$

$$=\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$$

$$=\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 1024 & -2048 \end{pmatrix}$$

$$=\begin{pmatrix} 3070 & -6138 \\ 1023 & -2045 \end{pmatrix}$$

(d) (i) x_n remains on the line x-3y=0 but with a scaling factor of 2 on each iteration.
(ii) $x_n = x_0$ for all n.

Qn.19
(a) $g(x)=d/dx\{x.\exp(4-x^2)\} - 1$
$= \exp(4-x^2)\{1-x.(2x)\}-1$
$= (1-2x^2)\exp(4-x^2)-1$
(b) (i) g(0)=exp(4)-1>0;
g(1)=-1.exp(3) – 1 <0
g is continuous so g(x) has a root between 0 and 1.

(ii) $g'(x)= \exp(4-x^2)\{-2x.(1-2x^2)-4x\}$
$= \exp(4-x^2)(\{4x^2-6x\}$

$$x_{n+1}=x_n-\frac{g(x_n)}{g'(x_n)}$$

$$= x_n-\frac{(1-2x_n^2)\exp(4-x_n^2)-1}{(4x_n^3-6x_n)(\exp(4-x_n^2)}$$

(c) $0.6965(\exp(4-0.6965^2)-1)$
= 22.7 to 1 dec.pl.

(d) Area under graph=

$$\int_0^2 x.\exp(4-x^2)dx - \int_0^2 x\ dx$$

$$=\left[-\frac{1}{2}\exp(4-x^2)\right]_0^2-\left[\frac{1}{2}x^2\right]_0^2$$

= (-½ + ½exp(4)) – (2 – 0)

= 26.79-2 = 24.8 to 1 dec. pl.

(e) graph is approximately half the rectangle 2 x max(y), i.e. is approximately max(y).

Qn.20 (a) (i) 3
(ii) it must not be a multiple of 3

(b) 27=3.8 + 3
8=2.3 + 2
3=1.2 + 1
we now have an equation for 1 so work backwards:
1=3 - 1.2
=3 – 1(8-2.3) = -1.8+3.3
=-1.8 +3(27 – 3.8)
=-1.8 –9.8 in modulo 27 arith
=-10.8 = 17.8 in mod 27 arith
so inverse of 8 is 17.

(c) 17.25=425=20(mod 27)
17.10=170= 8(mod 27)
17. 9=153=18(mod 27)
17.13=221= 5(mod 27)
so message is THREE.