Question 20 *(H5:2; B5.2.2, 5.2.3)*

(a) $P(X \leq 40) = \Phi\left(\dfrac{40-36}{\sqrt{8}}\right) = \Phi(1.41) = 0.921.$ [2]

(b) The 15% quantile is given by $c = \mu_X + q_{0.15}\sigma_X = 36 - 1.036\sqrt{8} = 33.1.$ [3]

Question 21 *(H5:4; B5.2.4)*

$W_1 \sim N(102, 5)$, $W_2 \sim N(103, 7)$ so $W_2 - W_1 \sim N(103 - 102, 7 + 5) = N(1, 12)$; so

$$P(W_2 - W_1 > 2) = 1 - \Phi\left(\frac{2-1}{\sqrt{12}}\right) = 1 - \Phi(0.29) = 0.386.$$ [6]

Question 22 *(H2:3, 5:6; B2.3.2, 5.5.1)*

The number of defective fuses in the batch is $B(1000, 0.02)$; by the central limit theorem, this is approximately normal $N(1000 \times 0.02, 1000 \times 0.02 \times 0.98) = N(20, 19.6)$. [Note: no probability calculations are required.] [3]

Question 23 *(H5:1, 5:2; B5.2, 5.3)*

The normal distribution is symmetric, unimodal and $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \simeq 1$. (You may have chosen other properties.) [3]

Question 24 *(H6:2, 6:5, S4; B6.4.1)*

(a) $E(X) = \dfrac{K\theta}{\theta - 1}$, so, for the moment estimator,

$$\overline{X} = \frac{K\widehat{\theta}_{MM}}{\widehat{\theta}_{MM} - 1}.$$

Solving for $\widehat{\theta}_{MM}$, we have

$$\widehat{\theta}_{MM} = \frac{\overline{X}}{\overline{X} - K}.$$ [2]

(b) $\overline{x} = \dfrac{10\,940}{50} = 218.8$, so $\widehat{\theta}_{MM} = \dfrac{218.8}{218.8 - 200} = \dfrac{218.8}{18.8} = 11.64.$ [1]

(c) $\widehat{\theta}_{ML} = \dfrac{n}{\sum \log(x_i/K)} = \dfrac{n}{\sum \log x_i - n \log K} = \dfrac{50}{269.32 - 50 \log 200} = 11.35.$ [2]

(d) The estimates are very close to each other, the difference being only 0.29 (about $2\frac{1}{2}$%). With a reasonable sample size they might be expected to be similar. [2]

Question 25 *(H7:3, 7:5; B7.3.1)*

(a) If the set of experiments were repeated, then about 95 times out of 100 the calculated interval would contain the true value. The given interval is that obtained from the single set which actually was carried out. [2]

(b) The value would be 299 852.2: this is half-way between the two ends of the interval (based on a normal approximation). [1]

(c) This could be due to random variation (the one case in 20!). Or it could be because the figures or estimate are biased in some way. [2]