Question 12

What are the mean and variance of the discrete probability distribution given below?

x	-1	0	1
$p(x)$	$1/3$	$1/6$	$1/2$

[3]

$E(X) = \sum p(x)\,x = 1/6$

$E(X^2) = 1/3 \times 1 + 1/2 = 5/6$

$V(X) = 0.132$

$5/6 - (1/6)^2 = \frac{29}{36}$

$E(X) = -1 \times \frac{1}{3} + 0 \times \frac{1}{6} + 1 \times \frac{1}{2} = \frac{1}{6}$

$E(X^2) = \frac{1}{3} \times 1 + 0 \times \frac{1}{6} + 1 \times \frac{1}{2} = \frac{5}{6}$

$V(X) = E(X^2) - (E(X))^2 = \frac{5}{6} - \frac{1}{36} = \frac{29}{36}$

Question 13

The proportion of defective products in a factory producing ball bearings is 0.06. All bearings are checked automatically. Let the random variable X denote the number of bearings that are checked up to and including the first defective bearing one morning. What are $E(X)$ and $V(X)$? (You may assume that the defectiveness or otherwise of ball bearings forms a sequence of independent Bernoulli trials.) [2]

$Pr(n) = q^{n-1} p$

$E(X) = 1/p = 1/0.06 = 16.67$

$V(X) = \frac{q}{p^2} = \frac{0.94}{0.0036}$

$E(X) = \frac{1}{p} = \frac{1}{0.06} = 16.67 \sim 17.$

$V(X) = \frac{q}{p^2} = \frac{1-0.06}{0.06^2} = \frac{0.94}{0.0036} = 261.1$