Question 24

A standard probability model used to model variation in wages is the Pareto distribution with parameters K and θ, having c.d.f.

$$F(x) = 1 - \left(\frac{K}{x}\right)^{\theta} \quad x \geq K.$$

(a) Given the sample mean, $\overline{X}$, show that the moment estimator for θ is given by $\overline{X}/(\overline{X} - K)$.

A European company has to pay a statutory minimum wage of $K = 200$ Ecu. A random sample of 50 employees yielded the data $\sum x_i = 10\,940$, $\sum \log x_i = 269.32$.

(b) Calculate the moment estimate of θ using the data above.

(c) Use the data to calculate the maximum likelihood estimate of θ.

(d) Compare the estimates you obtained in parts (b) and (c). If there is a marked difference between them explain why this does not come as a surprise; if they are similar, explain why this might be expected. [7]

a) $1 = \quad \mu = \int_K^\infty x f(x)\,dx = \frac{K\theta}{\theta - 1}$

$\int x \frac{\theta K^\theta}{x^{\theta+1}}\,dx$

$\int_K^\infty \frac{\theta K^\theta}{x^\theta}\,dx$

$\int x^2 = \frac{x^3}{3}$

$\mu\theta - \mu = K\theta$

$\theta(\mu - K) = \mu$

$\theta = \frac{\mu}{\mu - K}$

$\left[\frac{\theta K^\theta}{\theta - 1 x^{\theta-1}}\right]_K^\infty = \frac{\theta K}{\theta - 1} = \overline{X}$

$\theta K = \overline{x}(\theta - 1)$

$\theta K = \overline{x}\theta - \overline{x}$

$\theta(K - \overline{x}) = -\overline{x}$

$\theta = \frac{\overline{x}}{\overline{x} - K}$

b) $\overline{x} = \frac{10940}{50} = 218.8$

$\theta = \frac{218.8}{218.8 - 200} = \frac{218.8}{18.8} = 11.64$

c) $\theta = \frac{n}{\sum \log(x_i/K)} = \frac{50}{\sum \log(x_i) - \sum \log K}$

$= \frac{50}{269.32 - 264.92} = 11.36$