PART TWO

Questions 17 to 24 carry a total of 32 marks.

Question 17

The fax machines of a merchant bank receive messages at a rate of two per minute. You may assume that such messages are independent of each other and occur randomly during a working day.

Give the distribution of each of the random variables defined below:

(a) A: the number of fax messages received between 10.00 am and 10.02 am;

(b) B: the time interval in minutes between successive fax messages. [4]

Poisson $P(x) = e^{-\mu}\mu^x$ / $x!$

exponential $f(t) = \lambda e^{-\lambda t}$

Poisson$(4) = e^{-4}4^x$ / $x!$

$\lambda t = 2$

$M(2)$

$F(t) = 1 - e^{-\lambda t}$

Question 18

If X is a Poisson random variable with mean 2.4, calculate the probability that X is no greater than 1. [2]

$P(0) + P(1) = e^{-2.4} \times 2.4^0 + e^{-2.4} \times 2.4$ / $0!$ $1!$

$= 0.0907 + 0.2177 = 0.3084$

Question 19

(a) Why would one doubt the wisdom of fitting a Poisson model to a sample that had a mean of 3 and a variance of 9?

(b) If X, Y and Z are three independent Poisson variates, with means μ_X, μ_Y, and μ_Z respectively, what is the variance of $X + Y + Z$? [2]

a) For a Poisson model, the mean should equal the variance.

b) $V(X+Y+Z) = \mu_X + \mu_Y + \mu_Z$