

Question 18

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The independent random variables X , Y and Z are exponential with means 1, 2, and 6. What is the mean and variance of their sum?

[3]

$$\begin{aligned} E(X+Y+Z) &= E(X) + E(Y) + E(Z) \\ &= 1 + 2 + 6 = 9 \\ V(S) &= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \\ &= \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} \\ &= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{6^2} = 1 + \frac{1}{4} + \frac{1}{36} = \frac{36 + 9 + 4}{36} = \frac{49}{36} \end{aligned}$$

Question 19

An agricultural biologist was interested in the mean number of ripe kernels per head of wheat in a particular field. This was to be estimated by choosing a random sample of heads of wheat from the field, counting the kernels on each, and calculating the sample mean. Explain briefly, in statistical terms, why it is better to use a large sample than a small sample in this context.

[3]

The ~~more~~ confidence interval for the mean of a random sample is narrower for larger samples. The variance, hence scatter from the 'true distribution' whatever it may be, is smaller for a large sample.

Question 20

Assuming that the variation in heights for adult males in the British population may be adequately modelled by a normal distribution with mean 173 cm and standard deviation 6.0 cm, find the proportion of adult male Britons who are at least 180 cm tall.

[3]

$$\frac{180 - 173}{6} = 1.167$$

$$P(H \geq 180) = 1 - 0.8784 = 0.1216$$