

PART II

- (i) You should attempt at most TWO questions in this part.
 (ii) Each question in this part carries 18 marks.

Question 9

- (a) Use the Cauchy-Riemann Theorem and its converse to determine the point, or points, of \mathbb{C} at which the function

$$f(z) = \bar{z}(1 - z)$$

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is differentiable.

- (b) Let g be the function $g(z) = \frac{1}{z^2}$.

- (i) Show that g is conformal at i .

- (ii) Let Γ_1 and Γ_2 be the paths

$$\Gamma_1 : \gamma_1(t) = e^{it} \quad (t \in [0, 2\pi]),$$

$$\Gamma_2 : \gamma_2(t) = (1 - t)i - t \quad (t \in \mathbb{R}).$$

Show that Γ_1 and Γ_2 meet at the point i , and sketch Γ_1 and Γ_2 on the same diagram.

- (iii) Describe the effect of g on a small disc centred at i and hence make a sketch showing the approximate directions of the paths $g(\Gamma_1)$ and $g(\Gamma_2)$ near the point $g(i)$.

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Question 10

- (a) Let f be the function $f(z) = \frac{1}{z(z-3)}$.

- (i) Locate and classify the singularities of f .

- (ii) Find the Laurent series about 1 for f on the annulus $\{z : 1 < |z-1| < 2\}$, giving the constant term and two terms on each side of it.

[9]

- (b) (i) Find the Taylor series about 0 (up to the term in z^4) for the function

$$g(z) = \exp(\cos z - 1),$$

and explain why the series represents g on \mathbb{C} .

- (ii) Hence evaluate the integral

$$\int_C z^3 g(1/z) dz,$$

where C is the circle $\{z : |z| = 2\}$.

[9]