

Question 5

- (a) Find the residues of the function

$$f(z) = \frac{z}{(3z^2 - 1)(z^2 - 3)}$$

at those poles which lie inside the unit circle $C = \{z : |z| = 1\}$.

[3]

- (b) Hence evaluate the real trigonometric integral

$$\int_0^{2\pi} \frac{1}{1 + 3 \sin^2 t} dt.$$

[5]

Question 6

Use Rouché's Theorem to determine the number of solutions of the equation

$$z^7 + 5z^3 + 7 = 0$$

in the annulus $\{z : 1 < |z| < 2\}$.

[8]

Question 7Let $q(z) = \bar{z}^2$ be a velocity function.

- (a) Explain why
- q
- represents a model fluid flow.

[1]

- (b) Determine a stream function for this flow. Hence sketch the streamline through the point
- $e^{i\pi/3}$
- , and indicate the direction of flow.

[5]

- (c) Find the flux of
- q
- across the unit circle.

[2]

Question 8

- (a) Prove that the iteration sequence

$$z_{n+1} = z_n(1 - z_n), \quad n = 0, 1, 2, \dots,$$

with $z_0 = \frac{1}{2}$, is conjugate to the iteration sequence

$$w_{n+1} = w_n^2 + 1/4, \quad n = 0, 1, 2, \dots,$$

with $w_0 = 0$.

[2]

- (b) Which of the following points
- c
- lie in the Mandelbrot set:

(i) $c = \frac{1}{2}i$,

(ii) $c = 1 + \frac{1}{2}i$?

Justify your answer in each case.

[6]