

M337 Solutions to Specimen Examination Paper

This is a rough guide to the type of written solutions required. We do not expect your solutions to be as neatly laid out as these and, of course, for some questions there are alternative ways of doing them. Any correct method receives full marks.

A mark scheme is provided so that you can mark your own attempts. This uses accuracy marks (A-marks) and method marks (M-marks), with some indication about how these are awarded. Note that results used may be referred to by name, handbook (HB) reference or unit reference.

A few comments are included to try to help you with your revision and examination technique. In general, we recommend that you draw diagrams where relevant, as this should help you in devising solutions.

Solutions to Part I

Mark Scheme

Question 1

- (a) Since $w = 2(1 - i)/2 = 1 - i$, we have

$$\text{Arg } w = -\pi/4.$$

1A

1A

- (b) Since $w = \sqrt{2}e^{-i\pi/4}$, the cube roots of w are

$$z_k = (\sqrt{2})^{1/3} \left(\cos\left(-\frac{\pi}{12} + k\frac{2\pi}{3}\right) + i \sin\left(-\frac{\pi}{12} + k\frac{2\pi}{3}\right) \right), \quad k = 0, 1, 2;$$

1A

1M

that is,

$$z_0 = 2^{1/6}(\cos(-\pi/12) + i \sin(-\pi/12)),$$

$$z_1 = 2^{1/6}(\cos 7\pi/12 + i \sin 7\pi/12),$$

$$z_2 = 2^{1/6}(\cos 5\pi/4 + i \sin 5\pi/4).$$

 2A ($-\frac{1}{2}$ for each error)

(The form $z_0 = 2^{1/6}e^{-i\pi/12}$, etc., is also acceptable.)

Since $-\pi/4$ is the principal argument of w , z_0 is the principal cube root of w .

1A

- (c) $n = 4$ (since $\text{Arg } w = -\pi/4$).

1A

Question 2

	$A \cup B$	$A \cap D$	$D - \{2\}$	∂B
(a) region	✓	×	✓	×
(b) closed	×	×	×	✓

1A for each answer

(Here you probably found that sketching the given sets A , B and D helped you to answer the question.)