(iii) We choose a Möbius transformation whose extension to $\widehat{\mathbb{C}}$ maps i, $-i$ (which are inverse points with respect to $\partial\mathcal{R}_2$) to 0, ∞ (which are inverse points with respect to $\partial\mathcal{S}$): 1M

$$w = \frac{z_2 - i}{z_2 + i}.$$ 1A

Since this maps 0 to $-1 \in \partial\mathcal{S}$, we deduce that it maps $\partial\mathcal{R}_2$ onto $\partial\mathcal{S}$ and hence $\mathcal{R}_2$ onto $\mathcal{S}$. 1M

(iv) Composing the above mappings, we deduce that

$$f(z) = w = \frac{z_2 - i}{z_2 + i} = \frac{z_1^2 - i}{z_1^2 + i} = \frac{\left(\dfrac{z+i}{-z+i}\right)^2 - i}{\left(\dfrac{z+i}{-z+i}\right)^2 + i}$$ 1M, $\frac{1}{2}$A

is a one-one conformal mapping (because each of the constituent mappings is one-one and conformal) from $\mathcal{R}$ onto $\mathcal{S}$. The corresponding inverse function is

$$f^{-1}(w) = z = \frac{iz_1 - i}{z_1 + 1} = \frac{i\sqrt{z_2} - i}{\sqrt{z_2} + 1} = \frac{i\sqrt{\dfrac{iw+i}{-w+1}} - i}{\sqrt{\dfrac{iw+i}{-w+1}} + 1},$$ $1\frac{1}{2}$M, 1A

where the formula for the inverse function of a Möbius transformation has been used twice.

(This is another question where drawing diagrams will have helped you answer the question.)