

PART I

- (i) You should attempt as many questions as you can in this part.
(ii) Each question in this part carries 8 marks.

Question 1

Determine each of the following complex numbers in Cartesian form, simplifying your answers as far as possible.

- (a) $(1+i)^4$ [2]
(b) $\cos(\pi - i \log_e 2)$ [3]
(c) $(-e)^{i\pi}$ [3]

Question 2

Let $A = \{z : 1 \leq |z-i| \leq 2\}$ and $B = \{z : -\pi/4 < \text{Arg } z < \pi/4\}$.

- (a) Make separate sketches of the sets A and B . [2]
(b) For each of the sets A , B and $C = \text{ext } A$
(i) state whether or not it is a region, and if it is not a region, then explain why not;
(ii) state whether or not it is compact, and if it is not compact, then explain why not. [6]

Question 3

- (a) Evaluate

$$\int_{\Gamma} \text{Im } z \, dz,$$

where Γ is the line segment from i to 1 .

[4]

- (b) Determine an upper estimate for the modulus of

$$\int_C \frac{z^2-1}{z^2+1} dz,$$

where C is the circle $\{z : |z| = 2\}$.

[4]

Question 4

- (a) Evaluate the following integrals, where C is the circle with centre i and radius 2. Name any standard results that you use and check that their conditions hold.

(i) $\int_C \frac{e^{i\pi z}}{z+1} dz$

(ii) $\int_C \frac{e^{i\pi z}}{z+2} dz$

[6]

- (b) Use Liouville's Theorem to establish that there is a complex number z such that

$$|\cos(1+z^2)| > 100.$$

[2]