

Question 5

- (a) Find the residues of the function

$$f(z) = \frac{z^2 + 1}{z(z - \frac{1}{2})(z - 2)}$$

at each of the poles of f .

[4]

- (b) Hence evaluate the integral

$$\int_0^{2\pi} \frac{\cos t}{5 - 4 \cos t} dt.$$

[4]

Question 6

- (a) Use Rouché's Theorem to show that the equation

$$2z^3 + 5z - 1 = 0$$

has three solutions inside the circle $C_1 = \{z : |z| = 2\}$, exactly one of which lies inside the circle $C_2 = \{z : |z| = 1\}$.

[7]

- (b) Show that the solution inside
- C_2
- is real and positive.

[1]

Question 7Let $q(z) = iz$ be a velocity function.

- (a) Explain why
- q
- represents a model fluid flow on
- \mathbb{C}
- .

[1]

- (b) Determine a stream function for this flow. Hence find the equations of the streamlines through the points
- i
- and
- $1 + i$
- , and sketch these streamlines indicating the direction of flow.

[6]

- (c) Determine the flux of
- q
- across the path
- Γ
- , where

$$\Gamma : \gamma(t) = (1 + i)t \quad (t \in [1, 2]).$$

[1]

Question 8

- (a) Show that
- i
- is an indifferent fixed point of the function

$$f(z) = z^2 - iz + i.$$

[3]

- (b) Show that

(i) the point $1 + i$ does not lie in the Mandelbrot set;(ii) the point $-\frac{9}{10} - \frac{\sqrt{3}}{10}i$ does lie in the Mandelbrot set.

[5]