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then by Weierstrass's theorem
 f is analytic on $\mathbb{D}: |z| \leq 13$.

d) i) $\Gamma(2i+3) = (2i+2)\Gamma(2i+2)$ by the functional equation of the gamma function.
 $= (2i+2)(2i+1)\Gamma(2i+1)$
 $= (2i+2)(2i+1)(2i)\Gamma(2i)$
 $= (2i+2)(2i+1)(2i)(2i-1)\Gamma(2i-1)$
 $= \frac{\Gamma(2i+3)}{\Gamma(2i-1)} = (2i+2)(2i+1)2i(2i-1)$
 $= (-4+2+6i)(-4-2i)$
 $= (-2+6i)(-4-2i)$
 $= 8+12+4i-24i = 20-20i$

3A, 11

or $\text{ANS} = 20 - 20i$ ✓

ii) $\int_C \Gamma(z) dz = 2\pi i$ (Sum of residues of $\Gamma(z)$ inside C)
 $= 2\pi i (\text{Res}(\Gamma, 0) + \text{Res}(\Gamma, -1) + \text{Res}(\Gamma, -2))$ ✓

3A, 14 $= 2\pi i \left(\frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} \right)$

2A $= 2\pi i \left(1 - 1 + \frac{1}{2} \right) = \pi i$ ✓

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