TMA M337 03 **Cut-off date** 4 August 1999

Question 1 (*Unit C1*) – 33 marks

(a) Let

$$f(z) = \frac{z^2+1}{3z^3+10iz^2-3z}.$$

(i) Determine the residue of f at each of its poles. [5]

(ii) Deduce, by the strategy in Section 2, that

$$\int_0^{2\pi} \frac{\cos t}{5+3\sin t}\,dt = 0.$$ [5]

(b) Use a theorem from Section 3 to show that

$$\int_{-\infty}^{\infty} \frac{t^2\cos(2t)}{(t^2+4)^2}\,dt = -\frac{3\pi e^{-4}}{4}.$$ [11]

(c) Use a method given in Section 4 to determine the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{16n^2-1}.$$ [12]

Question 2 (*Unit C2*) – 33 marks

-2
-1 - i
3 - 3i

(a) By using a suitable function Arg_ϕ, determine a continuous argument function for the path

$$\Gamma : \gamma(t) = 2t+1-i(t+2) \quad (t \in [-1,1]),$$

and hence evaluate $\mathrm{Wnd}(\Gamma, 0)$. [4]

(b) Determine the number of zeros of the function

$$f(z) = z^4 + 6iz^2 + z - 2i$$

in each of the following sets.

(i) $S_1 = \{z : |z| < 3\}$ [4]

(ii) $S_2 = \{z : 2 < |z| < 3\}$ [6]

(iii) $S_3 = \{z : |z| \geq 3\}$ [2]

(c) Let $f(z) = e^z + z - 1$.

(i) Determine the set of points α in $\mathbb{C}$ such that the function f fails to be one-one near α. [2]

(ii) Invert the Taylor series about 0 for f, giving the first three non-vanishing terms. [7]

(d) Determine

$$\max\{|\exp(2+iz^2)| : |z| \leq 2\},$$

and find the point(s) at which the maximum is attained, giving your answer in Cartesian form. [8]