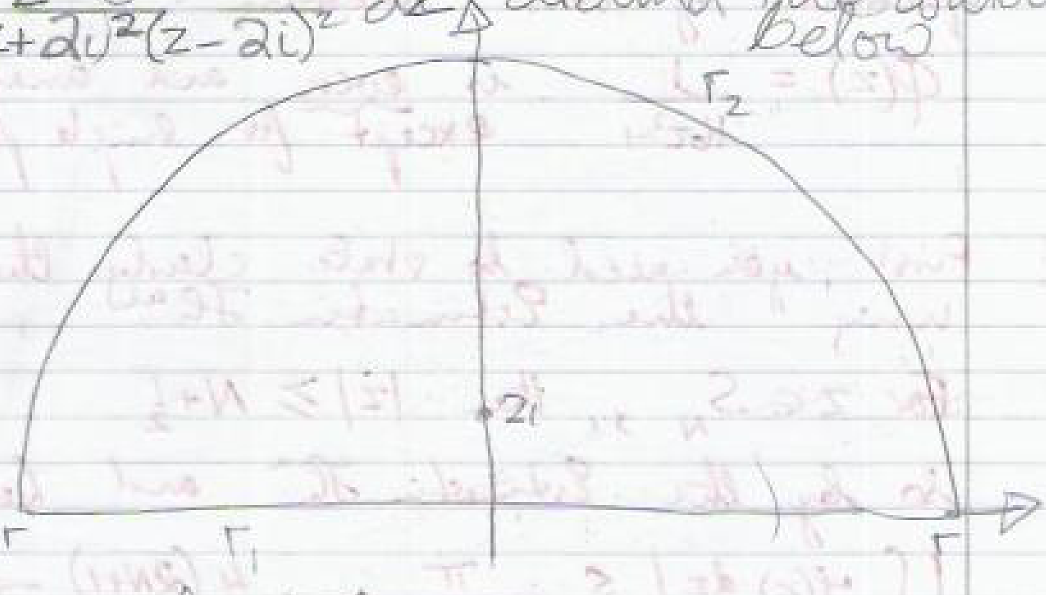
b) $\int_{-\infty}^{\infty} \frac{t^2 \cos 2t}{(t^2+4)^2}\,dt$

consider the integral

$= \int \frac{z^2 e^{2zi}}{(z+2i)^2(z-2i)^2}\,dz$ around the contour below

See Note!



2A,M

By the residue theorem

1M $*\ \int_{-r}^{r} \frac{z^2 e^{2iz}\,dz}{(z+2i)^2(z-2i)^2} + \int_{\Gamma_2} \frac{z^2 e^{2iz}\,dz}{(z+2i)^2(z-2i)^2} = 2\pi i \operatorname{Res}(f, 2i)$ ✓

where Γ_2 is the semicircle radius r

Since the only residue of $\frac{t^2 e^{2ti}}{(t^2+4)^2}$ consider

is at $\cancel{t}$ $z = 2i$. ✓

$\operatorname{Res}(f, 2i) = \frac{1}{1!}\lim_{z \to 2i} \left(\frac{d}{dz}\left(\frac{z^2 e^{2iz}}{(z+2i)^2}\right)\right)$ ✓

$= \lim_{z \to 2i} \left(\frac{2z e^{2iz}}{(z+2i)^2} + \frac{2iz^2 e^{2iz}}{(z+2i)^2} - \frac{2z^2 e^{2iz}}{(z+2i)^3}\right)$ ✓

$= \frac{4i e^{-4}}{(4i)^2} + \frac{2i(2i)^2 e^{-4}}{(4i)^2} - \frac{2(2i)^2 e^{-4}}{(4i)^3}$ ✓

$= \frac{-ie^{-4}}{4}$ ✓ $+ \frac{ie^{-4}}{2}$ ✓ $+ \frac{ie^{-4}}{8}$ ✓

3A,M $= \frac{ie^{-4}}{8}\left(-2+4+1\right) = \frac{3ie^{-4}}{8}$ ✓

ie $\operatorname{Res}(f, 2i) = \frac{3ie^{-4}}{8}$

1M Use the estimation theorem on 2nd integral in $*$

$\int_{\Gamma_2} \frac{z^2 e^{2iz}}{(z+2i)(z-2i)}\,dz = \int_{\Gamma_2} \frac{z^2 e^{2iz}}{(z^2+4)^2}\,dz$

$\left|\frac{z^2 e^{2iz}}{(z^2+4)^2}\right| = \left|\frac{z^2 e^{2i(x+iy)}}{(z^2+4)^2}\right| = \left|\frac{z^2 e^{2ix} e^{-2y}}{(z^2+4)^2}\right|$