$\le \left|\dfrac{r^2}{(r^2-4)^2}\right|$ since $|e^{-2y}|\le 1$ and $|e^{2ix}|=1$

2A,M $\left|\int_{C_2} \dfrac{z^2 e^{2iz}}{(z^2+4)^2}\,dz\right| \le \left|\dfrac{r^2}{(r^2-4)^2}\right| \times \pi r \to 0$ ✓

as $r \to \infty$ ∴ the integral equals 0.

From *, as $r \to \infty$: $\therefore \int_{-\infty}^{\infty} \dfrac{t^2 e^{2it}}{(t^2+4)^2}\,dt = 2\pi i \times \dfrac{3ie^{-4}}{8} = \dfrac{-3\pi e^{-4}}{4}$ ✓

$$\int_{-\infty}^{\infty} \frac{t^2 \cos 2t}{(t^2+4)^2}\,dt + i\int \frac{t^2 \sin 2t}{(t^2+4)^2}\,dt = \frac{-3\pi e^{-4}}{4}$$

From which we see that the imaginary part is zero, and

[11] 2A,M $\int_{-\infty}^{\infty} \dfrac{t^2 \cos 2t}{(t^2+4)^2}\,dt = \dfrac{-3\pi e^{-4}}{4}$ ✓

c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{16n^2-1} = \sum \dfrac{(-1)^n}{16(n-1/4)(n+1/4)}$ ✓

See Note 2

The series is equivalent to finding the sum of the residues of

$$f(z) = \frac{\pi}{16(z-1/4)(z+1/4)\sin \pi z}$$

at the poles of $\dfrac{1}{16(z-1/4)(z+1/4)}$ and zero

By Thm 4.2:

$\sum \dfrac{(-1)^n}{16n^2-1} = \dfrac{-1}{2}\left(\mathrm{Res}(f,0) + \sum \mathrm{Res}(f,\alpha_i)\right)$ once the condition of $\int_{S_N} \to 0$ has been checked.

2A,M / 3 The only poles of $\dfrac{1}{16(z-1/4)(z+1/4)}$ are at $z = \pm 1/4$. ✓ See Note 2

$\mathrm{Res}(f, 1/4) = \dfrac{\pi}{16(1/4+1/4)\sin \pi/4} = \dfrac{\pi}{8\sqrt{2}/2} = \dfrac{\pi}{4\sqrt{2}}$ ✓

3A $\mathrm{Res}(f, -1/4) = \dfrac{\pi}{16(-1/4-1/4)\sin(-\pi/4)} = \dfrac{\pi}{4\sqrt{2}}$ ✓

1A and $\mathrm{Res}(f, 0) = \dfrac{\pi}{-1} \times \dfrac{1}{\pi\cos(0)} = -1$ ✓ by g/h' rule.