

Note 4

One step missed here

From (i) f has 4 zeros in $\{z: |z| < 3\}$

(ii) f has 2 zeros in $\{z: |z| < 2\}$

$\therefore f$ has 2 zeros in $\{z: 2 \leq |z| < 3\}$

note

but f has no zeros on $|z| = 2$ since for $|z| = 2$

~~$|f(z)| > 0$~~ $|z^4 + z - 2i| < |6z^2|$ as shown previously.

Notes

Some explanation ^{required} here.

f is one-one near a if and only if $f'(a) \neq 0$.

Now, if $f(z) = e^z + z - 1$ etc

so let f point near which f fails to be one-one is $\{(2n+1)\pi i: n \in \mathbb{Z}\}$