2A $z = b_1(\beta) + b_2\beta^2 + b_3\beta^3 \ldots$

$= b_1\left(2z + \frac{z^2}{2!} + \frac{z^3}{3!} \cdots\right) + b_2\left(2z + \frac{z^2}{2!} + \frac{z^3}{3!} \cdots\right)^2$

$+ b_3\left(2z + \frac{z^2}{2!} + \frac{z^3}{3!} \cdots\right)^3 + \ldots$

$= 2b_1 z + z^2\left(\frac{b_1}{2} + 4b_2\right) + z^3\left(\frac{b_1}{6} + 2b_2 + 8b_3\right) + \ldots$

equating coefficients

$b_1 = \frac{1}{2}$ ✓

$\frac{b_1}{2} + 4b_2 = 0 \Rightarrow b_2 = -\frac{b_1}{8} = \frac{-1}{16}$ ✓

$\frac{b_1}{6} + 2b_2 + 8b_3 = 0 \Rightarrow b_3 = -\frac{1}{8}\left(\frac{b_1}{6} + 2b_2\right)$

$= -\frac{1}{8}\left(\frac{1}{12} - \frac{2}{16}\right)$

$= -\frac{1}{8}\left(\frac{16-24}{192}\right) = \frac{1}{192}$ ✓

3A $\therefore f^{-1}(w) = \frac{w}{2} - \frac{w^2}{16} + \frac{w^3}{192} + \cdots$ ✓.

$\boxed{\frac{7\frac{1}{2}}{9}}$

d) By the maximum principle the maximum is attained on the boundary, ✓ $|z|=2$ (since $\exp(2+iz^2)$ is analytic on $|z|<2$, and continuous on $\{z : |z| \leq 2\}$

3A,M Put $z = 2e^{it}$ ✓

$|\exp(2+iz^2)| = |\exp(2+4ie^{2it})|$

$= |\exp(2+4i(\cos 2t + i\sin 2t))|$ ✓

$= |\exp((2-4\sin 2t) + 4i\cos 2t)|$

$= \underbrace{|\exp(2-4\sin 2t)|}_{\text{max when}}\ \underbrace{|\exp(4i\cos 2t)|}_{\text{equals unity}}$ ✓.

2A $(2-4\sin 2t)$ max

ie when $\sin 2t = -1$ ✓

(or $t = 3\pi/4$ or $t = 7\pi/4$

then $|\exp(2-4\sin 2t)| = |\exp(2-4\times -1)|$

$= e^6$ ✓

when $t = \frac{3\pi}{4}$

$z = 2\exp(3\pi i/4)$

$= 2\cos\frac{3\pi}{4} + 2i\sin\frac{3\pi}{4}$

$\frac{7}{8}$ $2\frac{1}{3}$ A $\frac{26\frac{1}{2}}{33}$ $= -\sqrt{2} + i\sqrt{2}$ ✓ and when $t = 7\pi/4$

$z = \sqrt{2} - i\sqrt{2}$