these aren't in the interval $[0, 2\pi)$

These arguments are more consistent with Arg_{π} being used and not $\mathrm{Arg}_{2\pi}$ as you intended.

$\frac{3}{4}$ A,M $= \dfrac{(-\pi/4) - (-3\pi/4)}{2\pi} = \dfrac{1}{4}$ ✓

(b) i) $f(z) = z^4 + 6iz^2 + z - 2i$

Put $h(z) = z^4 \quad g(z) = 6iz^2 + z - 2i$

On $|z| = 3$, $|h(z)| = |z|^4 = 81$ ✓

$|g(z)| \le 6|3|^2 + |3| + 2 = 59$ ✓

$\therefore |h| \ge |g|$

$\therefore$ By Rouché's theorem f and h have the same number of zeros in $S_1 = \{z : |z| = 3\}$ $h(z) = z^4$ has zeros

4A,M on S_1 $\therefore$ f has four zeros in S_1. ✓

ii) Put $h(z) = 6iz^2$, $g(z) = z^4 + z - 2i$

Then on $|z| = 2$

$|h(z)| = 6 \times 2^2 = 24$ ✓

$|g(z)| \le |2|^4 + 2 + 2 = 20 < 24$ ✓

So $|h(z)| > |g(z)|$ on $|z| = 2$ ✓

4A,M $h(z)$ has two zeros in $D = \{z : |z| < 2\}$ ✓ By Rouché's theorem

$\therefore$ f has two zeros in D

$\frac{1}{2}$ A,M $\therefore$ f has $4 - 2 = 2$ zeros in the annulus S_2. See Note 4

iii) $f(z)$ is a polynomial order 4 $\therefore$ has ~~at most~~ exactly four zeros, but has 4 zeros inside $S_1 = \{z : |z| < 3\}$

$\boxed{\frac{11}{12}}$ $\therefore$ has exactly four zeros, all inside S_1 and none outside,

2A,M in S_3. ✓

c) i) $f'(z) = e^z + 1 = 0$ at $e^z = -1$, and $e^z = -1$ at $z = (2n+1)\pi i$ ✓

See note 5

$\frac{1\frac{1}{2}}{2}$ A,M $f''(z) = e^z \ne 0$ $\therefore$ at $(2n+1)\pi i = z$

f is two-one at these points ✓

ii) $f(z) = e^z + z - 1$

0/1 M $f(0) = 1 + 0 - 1 = 0$

1A $\therefore f^{-1}(0) = 0$ ✓

First note that since $f'(0) \ne 0$ then f is one-one near 0 and we can apply the strategy for inverting the Taylor series for f about 0.

Put $z = f^{-1}(\beta)$ where $\beta = e^z + z - 1$ and expand in a series ✓