1M $\int_0^\infty \frac{t^{2/3}}{t^2+1}\,dt = -\left(\pi e^{-\frac{2\pi i}{3}} \operatorname{cosec}\frac{2\pi}{3}\right)\left(\operatorname{Res}(f,i)+\operatorname{Res}(f,-i)\right)$

$= -\pi e^{-\frac{2\pi i}{3}} \operatorname{cosec}\frac{2\pi}{3}\left(\frac{e^{\pi i/3}-e^{-\pi i/3}}{2i}\right)$ ✓

$= \frac{-\pi}{\sin 2\pi/3}\left(\frac{e^{-\pi i/3}-e^{-5\pi i/3}}{2i}\right)$ $\left[e^{-\frac{5\pi i}{3}} = e^{\frac{\pi i}{3}}\right]$

$= \frac{+\pi}{2\sin\pi/3\cos\pi/3} \times \sin\pi/3$ ✓

$\frac{7}{9}$ 2A $= \frac{+\pi}{2\cos\pi/3} = +\pi$ ✓

c) $\sum_{n=1} \frac{e^{z^n}}{n^3+1}$ — Are you muddling the terms of the series with the series.

Given converges uniformly to zero. — No, eg when $z=0$, series is $\Sigma \frac{1}{n^3+1}$ which does not converge to 0.

$2\frac{1}{2}$/3 A,M $|z|\le 1 \Rightarrow |e^{z^n}| \le |e^{1^n}| = e$ ✓

$\therefore \left|\frac{e^{z^n}}{n^3+1}\right| \le \frac{e}{|n^3+1|}$ ✓ $\left[< \varepsilon\right.$ — we don't need this – it shows the terms $\to 0$ (P.T.O.)

for all n such that $n^3+1 > \frac{e}{\varepsilon}$

ie $\left.n > \sqrt[3]{\frac{e}{\varepsilon}-1}\right]$

$\sum_{n=1} \frac{e}{n^3+1} \le \sum_{n=1} \frac{e}{n^3} \le \sum_{n=1} \frac{e}{n^2} = \frac{e\pi^2}{6}$

By Weierstrass M-Test, the

$2\frac{1}{2}$/3 A,M series converges uniformly on $\{z: |z|\le 1\}$ ✓

(ii) z^n analytic on $\{z:|z|\le 1\}$

$\exp(z^n)$ analytic on $\{z:|z|\le 1\}$

$\frac{1}{n^3+1}$ analytic on $\{z:|z|\le 1\}$

$\therefore \frac{e^{z^n}}{n^3+1}$ is analytic on $\{z:|z|\le 1\}$

for each n $\therefore \left\{\frac{e^{z^n}}{n^3+1}\right\}$ is a sequence of functions analytic on $\{z:|z|\le 1\}$.

From c)i) they converge to the zero function. — True but we're interested in the unif. conv. of the series.