You've got most of the right ideas here but again you're not setting your solution out showing clearly how the conditions of the theorem being applied are satisfied.
With the Weierstrass M-test to be applied:
let

$$\varphi_n(z) = \frac{e^{z^n}}{n^3+1} \quad \text{for } n = 1, 2, \ldots \quad \text{and } E = \{z : |z| \leq 1\}.$$

Then for $z \in E$: $|\varphi_n(z)| = \frac{|e^{z^n}|}{n^3+1} < \frac{e^{|z^n|}}{n^3} \leq \frac{e}{n^3}$

So assumption 1 of W M-test holds with $M_n = e/n^3$.

$\sum_1^\infty M_n = e \sum_1^\infty \frac{1}{n^3}$ is convergent (we don't need its sum but you are correct in saying that it is less than $e\pi^2/6$)

So assumption 2 holds. $\therefore \sum_1 \varphi_n(z)$ is unif. convgt. on E.

(ii) Each of the partial sums $f_N(z) = \sum_{n=1}^{N} \varphi_n(z)$ is analytic in $R = \{z : |z| < 1\}$ and the functions f_N converge unif. to $f(z) = \sum_{n=1}^{\infty} \varphi_n(z)$ on each closed disc in R. Then, it follows from Weierstrass' Thm that $f(z) = \sum_{n=1}^{\infty} \varphi_n(z)$ is analytic on R.