Note 1: There is no need to work from first principles – you can use Th^m 3·4:

With $p(z) = z^2$ and $q(z) = (z^2+4)^2$. Then conditions 1 & 2 of Th^m 3.4 are satisfied. The singularities of $f(z) = \frac{p(z)}{q(z)} e^{2iz}$ are poles of order 2 at $\pm 2i$.

$$\therefore \; I = \int_{-\infty}^{\infty} \frac{t^2 e^{2it}}{(t^2+4)^2}\, dt = 2\pi i S + \pi i T$$

where $S = \ldots\ldots = \operatorname{Res}(f, 2i)$

and $T = \ldots\ldots = 0$ as there are no poles on the real axis.

etc

Thus the theorem avoids you having to prove $\int_{\Gamma_2} \to 0$ as $r \to \infty$ every time as this is guarenteed provided the conditions of the theorem are satisfied.