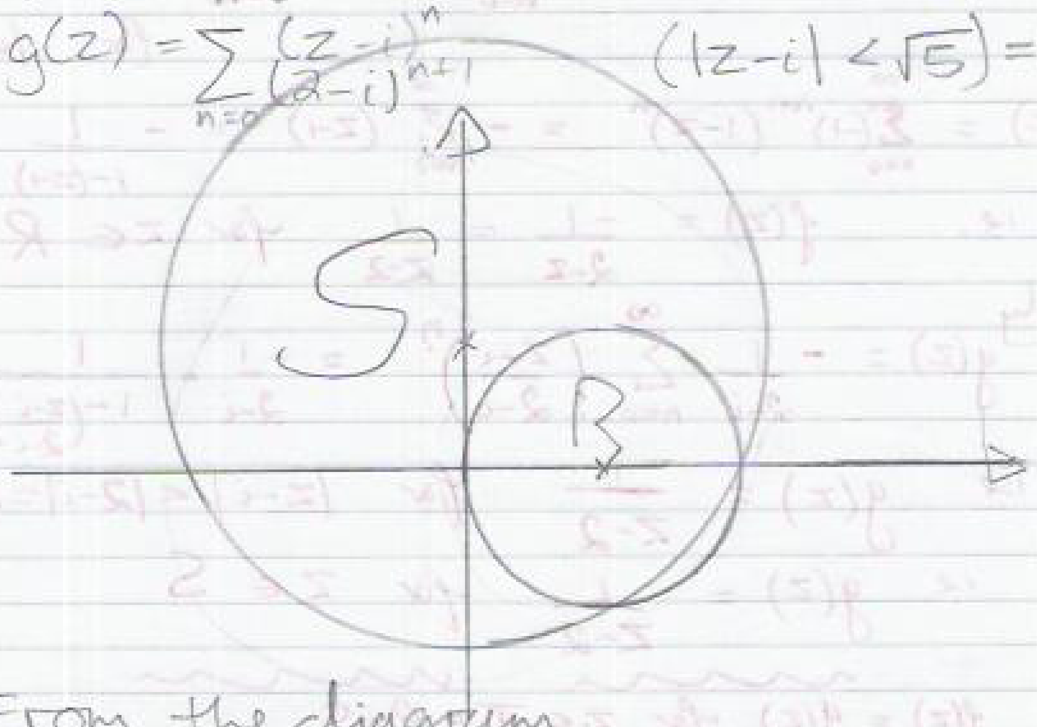
3) $f(z) = \sum_{n=0}^{\infty} (-1)^{n+1}(1-z)^n \quad (|z-1|<1) = R$

$g(z) = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}} \quad (|z-i|<\sqrt{5}) = S$




boundaries should be dotted.

From the diagram

$\{z:|z-1|<1\} \cap \{z:|z-i|<\sqrt{5}\} \subseteq S$ ✓

On R

$= -\sum(-1)^n(1-z)^n = -\sum(z-1)^n$ PT.0 1

$f(z) = \sum_{n=0} (-1)^{n+1}(1-z)^n = \frac{1}{1-(1-z)} = \frac{1}{2-z}$ $\frac{1}{z-2}$

$g(z) = -\sum_{n=0} \frac{(z-i)^n}{(2-i)^{n+1}} = \frac{1/(2-i)}{1-\frac{(z-i)}{(2-i)}} = \frac{1}{(2-i)-(z-i)}$

2/4 A,M

$= -\frac{1}{2-z} = \frac{1}{z-2}$ for $z \in S$

$f = g$ on $R \cap S$

3/4 A,M f and g are direct analytic continuations of each other PT.0 2

08/6

b) $\int_0^\infty \frac{t^{2/3}}{t^2+1}\,dt$ PT.0 3

1/3 A,M The integrand has no poles on +ve real axis and has poles $\pm i$ in $\mathbb{C}$

Write $\frac{z^{2/3}}{z^2+1} = \frac{e^{(2/3 \,\mathrm{Log}_{2\pi} z)}}{z^2+1} = \frac{e^{(2/3\,\mathrm{Log}\, z)}}{(z+i)(z-i)}$ Using g, h rule

$\mathrm{Res}(f,i) = \frac{\exp(2/3 \times \pi i/2)}{2i} = \frac{1}{2i}\exp(\pi i/3)$ ✓

3 A,M $\mathrm{Res}(f,-i) = \frac{\exp(2/3 \times 3\pi i/2)}{-2i} = \frac{-1}{2i}\exp(\pi i)$ ✓