

PART I

- (i) You should attempt as many questions as you can in this part.
- (ii) Each question in this part carries 8 marks.

Question 1

- (a) Let $w = \frac{1}{1+i}$.
 - (i) Determine $\text{Arg } w$.
 - (ii) Determine the principal fourth root of w in polar form. [5]
- (b) Determine the Cartesian form of $(-1)^{3i}$, simplifying your answer as far as possible. [3]

Question 2

Let $A = \{z : 1 \leq |z| \leq 2\}$ and $B = \{z : 0 < \text{Arg } z < \pi/2\}$.

- (a) Make separate sketches of the sets A , B and $B - A$. [3]
- (b) For each of the sets A , B and $B - A$
 - (i) state whether or not it is a region, and if it is not a region, then explain why not;
 - (ii) state whether or not it is compact, and if it is not compact, then explain why not. [5]

Question 3

Let Γ_1 be the line segment from 1 to i , and Γ_2 be the arc of the unit circle from 1 to i (anticlockwise). Evaluate the following integrals giving your answers in Cartesian form.

- (a) $\int_{\Gamma_1} \text{Re } z \, dz$ [4]
- (b) $\int_{\Gamma_1} \frac{1}{z} \, dz$ [3]
- (c) $\int_{\Gamma_2} \frac{1}{z} \, dz$ [1]

Question 4

Evaluate the following integrals naming any standard results that you use and checking that their required conditions hold.

- (a) $\int_{C_1} \frac{z^3}{z^2 + 2} \, dz$, where $C_1 = \{z : |z| = 1\}$ [2]
- (b) $\int_{C_2} \frac{z^3}{z^2 + 2} \, dz$, where $C_2 = \{z : |z| = 4\}$ [3]
- (c) $\int_{C_2} \frac{z^3}{(z+2)^2} \, dz$, where $C_2 = \{z : |z| = 4\}$ [3]