

Question 5

- (a) Find the residues of the function

$$f(z) = \frac{1}{z^3 - 1}$$

at each of the poles of f .

[4]

- (b) Hence evaluate the real improper integral

$$\int_{-\infty}^{\infty} \frac{1}{t^3 - 1} dt.$$

[4]

Question 6Let $D_0 = \{z : |z| < 2\}$ and $D_1 = \{z : |z| > 2\}$. Show that the analytic functions

$$f(z) = \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n \quad (z \in D_0)$$

and

$$g(z) = -\sum_{n=1}^{\infty} \left(\frac{2}{z}\right)^n \quad (z \in D_1)$$

are indirect analytic continuations of each other.

[8]

Question 7Let $q(z) = 1/z^2$ be a velocity function.

- (a) Explain why
- q
- represents a model fluid flow on
- $\mathbb{C} - \{0\}$
- .

[1]

- (b) Determine a stream function for this flow. Hence find the equation of the streamline through the point
- i
- , and sketch this, indicating the direction of flow.

[5]

- (c) Find the flux of
- q
- across the unit circle.

[2]

Question 8

- (a) Find the fixed points of the function
- $f(z) = 2z(1 - z)$
- and classify them as (super-)attracting, repelling or indifferent.

[3]

- (b) Which of the following points
- c
- lie in the Mandelbrot set:

(i) $c = -1 + i$;

(ii) $c = -1 - \frac{1}{8}i$?

Justify your answer in each case.

[5]