

Question 11

This question involves improper integrals which you may assume exist.

- (a) Give a brief reason why

$$\int_{-\infty}^{\infty} \frac{t}{t^4 - 1} dt = 0. \quad [1]$$

(b) Evaluate $\int_0^{\infty} \frac{1}{t^4 - 1} dt.$ [7]

(c) Evaluate $\int_0^{\infty} \frac{\sqrt{t}}{t^4 - 1} dt.$ [10]

Question 12

- (a) Determine the extended Möbius transformation \hat{f}_1 which maps i to 0, ∞ to 1 and $-i$ to ∞ . [3]

- (b) Let $R = \{z : |z - 1| < \sqrt{2}\} \cap \{|z + 1| < \sqrt{2}\}$, $S = \{z_1 : 3\pi/4 < \text{Arg}_{2\pi} z_1 < 5\pi/4\}$ and $T = \{w : \text{Re } w > 0\}$.

(i) Sketch the regions R , S and T .

(ii) Explain why $\hat{f}_1(R) = S$.

(iii) Hence determine a one-one conformal mapping f from R to T .

(iv) Determine a one-one conformal mapping g from R to the open unit disc $D = \{z : |z| < 1\}$. [There is no need to simplify your answer.] [15]

[END OF QUESTION PAPER]