

PART I

- (i) You should attempt as many questions as you can in this part.
- (ii) Each question in this part carries 8 marks.

Question 1

Determine each of the following complex numbers in Cartesian form, simplifying your answers as far as possible.

- (a) $\exp(1 + i\pi/6)$ [2]
- (b) $\frac{1}{(1-i)^4}$ [2]
- (c) $\text{Log } i$ [2]
- (d) $i^{(i/\pi)}$ [2]

Question 2

Let

$$A = \{z : 1 < |z - i| < 2\} \quad \text{and} \quad B = \{z : 0 \leq \text{Im } z \leq 2\}.$$

- (a) Make separate sketches of the sets A , B and $A - B$. [3]
- (b) Write down which of the sets A , B and $A - B$, if any, is
 - (i) open; A
 - (ii) a region; A
 - (iii) closed; B
 - (iv) compact. — [5]

Question 3

- (a) Determine the standard parametrization for the line segment Γ from 1 to i . [1]
- (b) Evaluate

$$\int_{\Gamma} \text{Re } z \, dz. \quad [2]$$

- (c) Determine an upper estimate for the modulus of

$$\int_{\Gamma} \frac{\cosh(\text{Re } z)}{4 + z^2} dz. \quad [5]$$

Question 4

Evaluate the following integrals, in which $C = \{z : |z| = 1\}$. Name any standard results that you use and check that their hypotheses are satisfied.

- (a) $\int_C \frac{1}{z^3} dz$ [3]
- (b) $\int_C \frac{\cos(z - \pi)}{z^3} dz$ [3]
- (c) $\int_C \frac{\cos z}{(z - \pi)^3} dz$ [2]