

Question 5

- (a) Find the residues of the function

$$f(z) = \frac{z^2 + 1}{z(2z + 1)(z + 2)}$$

at all its poles.

[3]

- (b) Hence evaluate the real integral

$$\int_0^{2\pi} \frac{2 \cos t}{5 + 4 \cos t} dt.$$

[5]

Question 6

Determine the number of zeros of the function

$$f(z) = z^3 + z - 3$$

in each of the following sets.

- (a)
- $\{z : 1 \leq |z| < 2\}$

[6]

- (b)
- $\{z : \operatorname{Im} z > 0\}$

[2]

Question 7Let $q(z) = i\bar{z}$ be a velocity function.

- (a) Explain why
- q
- represents a model fluid flow on
- \mathbb{C}
- .

[1]

- (b) Determine a stream function for this flow. Hence find the equations of the streamlines through the points 1 and
- i
- , and sketch these streamlines, indicating the direction of flow in each case.

[6]

- (c) Why is 0 neither a source nor a vortex?

[1]

Question 8

- (a) Prove that the iteration sequence

$$z_{n+1} = (z_n + 1)^2, \quad n = 0, 1, 2, \dots,$$

with $z_0 = -1$, is conjugate to the iteration sequence

$$w_{n+1} = w_n^2 + 1, \quad n = 0, 1, 2, \dots,$$

with $w_0 = 0$.

[2]

- (b) Find the fixed points of
- $P_1(z) = z^2 + 1$
- and determine their nature.

[3]

- (c) Show that
- $1 \notin M$
- and hence, or otherwise, determine whether or not 0 is in the keep set
- K_1
- and deduce the behaviour of the sequence
- $\{P_1^n(0)\}$
- .

[3]