

Question 5

- (a) Find the residues of the function

$$f(z) = \frac{1}{z^3 + 1}$$

at each of the poles of f .

[4]

- (b) Hence evaluate the real improper integral

$$\int_{-\infty}^{\infty} \frac{1}{t^3 + 1} dt.$$

[4]

Question 6

Use Rouché's Theorem to show that the equation

$$z^6 - 3iz^4 + 1 = 0$$

has exactly two solutions in the set $\{z : 1 < |z| < 2\}$.

[8]

Question 7Let $q(z) = \bar{z} + i$ be a velocity function.

- (a) Explain why
- q
- represents a model fluid flow.

[1]

- (b) Determine a stream function for this flow and hence find equations for the streamline through the point 1 and the streamline through the point
- $-1 + i$
- .

[4]

- (c) Sketch the streamlines found in part (b), showing the direction of flow, and also indicate any degenerate streamlines.

[3]

Question 8

- (a) Prove that the function
- $f(z) = z^2 + \frac{1}{4}$
- has exactly one fixed point, and that this fixed point is indifferent.

[3]

- (b) Determine which of the following points
- c
- lie in the Mandelbrot set.

(i) $c = -1 + i$.

(ii) $c = -\frac{1}{2} - \frac{1}{2}i$.

Justify your answer in each case.

[5]