

PART I

- (i) You should attempt as many questions as you can in this part.
 (ii) Each question in this part carries 8 marks.

Question 1

- (a) Determine each of the following complex numbers in Cartesian form, simplifying your answers as far as possible.

(i) $(1 - i)^8$

(ii) $(1 + i)^4$

[5]

- (b) Show that

$$\tan 2i = \left(\frac{e^4 - 1}{e^4 + 1} \right) i.$$

[3]

Question 2

Let

$$A = \{z : 1 \leq |z| \leq 2\} \quad \text{and} \quad B = \{z : |\operatorname{Arg} z| < 3\pi/4\}.$$

- (a) Make separate sketches of the sets A , B , $A - B$ and $A \cap B$.

[4]

- (b) For each of the sets $A - B$ and $A \cap B$, state whether the set is

(i) a region;

(ii) compact.

Give a brief reason in each case.

[4]

Question 3

Let Γ be the line segment from 0 to $1 + i$.

- (a) Evaluate

$$\int_{\Gamma} \bar{z}^2 dz.$$

[4]

- (b) Show that

$$\left| \int_{\Gamma} \exp(\bar{z}^2) dz \right| \leq \sqrt{2}.$$

[4]

Question 4

Find the Laurent series for the function

$$f(z) = \frac{1}{z^2 - 1}$$

- (a) about the point 1 on the punctured disc $\{z : 0 < |z - 1| < 2\}$;

[4]

- (b) about the point 0 on the region $\{z : |z| > 1\}$.

[4]

In each case, state the general term of the series.