Question 12

(a) (i) Determine the Möbius transformation f such that the extended Möbius transformation $\hat{f}$ maps 0 to $-i$, 1 to 1 and ∞ to i.

(ii) Show that $\hat{f}$ maps the extended imaginary axis to itself. [8]

(b) Let g be the function

$$g(z) = \sqrt{f(z)} \qquad (\operatorname{Re} z > 0),$$

where f is the function of part (a).

(i) Prove that g is a one-one conformal mapping and determine the image of g.

(ii) Obtain a formula for the rule of the inverse function g^{-1}. [10]

[END OF QUESTION PAPER]