

PART 1

- (i) You should attempt as many questions as you can in this part.
(ii) Each question in this part carries 8 marks.

Question 1

(a) Let $w = \frac{1}{-1+i}$.

- (i) Determine $\text{Arg } w$.
(ii) Determine the principal fourth root of w in polar form. [6]
(b) Determine the Cartesian form of $i^{(-i)}$, simplifying your answer as far as possible. [2]

Question 2

Let

$$A = \{z : |\text{Arg}(z-1)| < \frac{1}{2}\pi\} \quad \text{and} \quad B = \{z : |z| \leq 2\}.$$

- (a) Make separate sketches of the sets ∂A and $A \cup B$. [4]
(b) For each of the sets

$$A \cup \{1\} \quad \text{and} \quad A \cap B,$$

write down whether or not it is a region. [2]

- (c) For each of the sets

$$B - A \quad \text{and} \quad \overline{A},$$

write down whether or not it is a compact set. [2]

Question 3

Let f be the function $f(z) = \text{Im}(z^2)$.

- (a) Write down the domain of the function f and show that f is continuous. [4]
(b) Evaluate

$$\int_{\Gamma} f(z) dz,$$

where Γ is the line segment from 0 to $1+i$. [4]

Question 4

Let f be the function $f(z) = e^z \cos z$.

- (a) Find the Taylor series for the function f about 0 (up to the term in z^4). For what values of z does this Taylor series represent f ? Justify your answer. [4]
(b) Use your answer to part (a) to

- (i) evaluate

$$\int_C \frac{f(z)}{z^2} dz,$$

where C is the unit circle;

- (ii) determine the Taylor series about 0 (up to the term in z^3) for the function

$$g(z) = e^z (\cos z - \sin z). \quad [4]$$