M823 Solutions to the Specimen Examination Paper

Question 1

By the Euclidean algorithm,

$$144 = 1.99 + 45$$
$$99 = 2.45 + 9$$
$$45 = 5.9, \quad \text{so } (99, 144) = 9.$$ 2 for basic method

Working backwards, we have

$$9 = 99 - 2.45$$
$$= 99 - 2(144 - 99), \quad \text{so } 9 = 3.99 - 2.144. \qquad (1)$$ 3

Also, $176 = 19.9 + 5$

$$9 = 1.5 + 4$$
$$5 = 1.4 + 1$$
$$4 = 4.1, \quad \text{so } (9, 176) = 1.$$

Working backwards, we have

$$1 = 5 - 4$$
$$= 5 - (9 - 5) = 2.5 - 9$$
$$= 2(176 - 19.9) - 9, \quad \text{so } 1 = 2.176 - 39.9. \qquad (2)$$ 3

Combining (1) and (2) gives

$$1 = 2.176 - 39(3.99 - 2.144),$$

so $1 = -117.99 + 78.144 + 2.176,$

and $a = -117, b = 78, c = 2.$ 2

(Other answers are possible.)

[*Total: 10 marks*]

Question 2

If $n = 2^k p_1^{a_1} \dots p_r^{a_r}$, then $\phi(n) = 2^{k-1} p_1^{a_1 - 1} \dots p_r^{a_r - 1}(p_1 - 1) \dots (p_r - 1)$. 1

Since $\phi(n) = 18$, n can consist only of the primes 2, 3, 7 and 19. In fact, 3

$$n = 2^k 3^a 7^b 19^c, \text{ where } b, c = 0 \text{ or } 1.$$

If $c = 1$, then $n = 19m$, so $\phi(n) = \phi(19)\phi(m)$, giving

$\phi(m) = 1$ ~~or 2~~, so $n = \mathbf{19}$ or $\mathbf{38}$. 2

If $b = 1$, then $n = 7m$, so $\phi(n) = \phi(7)\phi(m)$, giving $\phi(m) = 3$, which is impossible. So $n = 2^k 3^a$. 2

If $k \geqq 1$, then $\phi(n) = 2^k 3^{a-1} = 18$, so $k = 1$, $a = 3$ and $n = \mathbf{54}$.

If $k = 0$, $\phi(n) = 2.3^{a-1} = 18$, so $a = 3$ and $n = \mathbf{27}$. Thus, $n = 19, 27, 38$ or 54. 2

[*Total: 10 marks*]

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