Question 11

Let p be the polynomial $t^3 - 7$ in $\mathbf{Q}[t]$.

(i) Find $\alpha, \beta \in \mathbf{C}$ such that $\mathbf{Q}(\alpha, \beta)$ is a splitting field for p over $\mathbf{Q}$. [4]

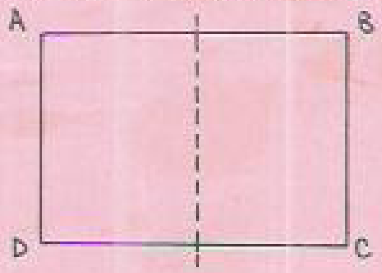
(ii) Find the degree $[\mathbf{Q}(\alpha, \beta) : \mathbf{Q}]$. [3]

(iii) Identify the Galois group $\Gamma(\mathbf{Q}(\alpha, \beta) : \mathbf{Q})$. [3]

Question 12 (Unit 13)

Determine whether or not the following ruler and compasses constructions are possible, given the line AB. Give a justification in each case.

(i) The rectangle $ABCD$ with the property that AB is larger than BC and if the rectangle is cut in half by a line parallel to BC then the two resulting smaller rectangles are the same shape as the original rectangle. [5]



(ii) A regular 204-gon with AB as one side. [5]

Question 13 (Unit 14)

Prove that the polynomial $f(t) = t^5 - 6t - 3$ in $\mathbf{Q}[t]$ is not soluble by radicals.

Your proof should contain references to any theorems used. [10]

Question 14 (Unit 15)

Let p and n be positive prime integers, and let $q = p^n$. Let K be a field of q elements.

(i) Let $\alpha \in K$. Show that the minimum polynomial of α over $\mathbf{Z}_p$ has degree 1 or n. [6]

(ii) Let $p = 2$, $n = 3$, $q = 8$, and let

$$K = \mathbf{Z}_2[t]/(t^3 + t + 1).$$

Find elements α and β in K whose minimum polynomials over $\mathbf{Z}_2$ have degrees 1 and 3 respectively. [4]

Question 15 (Unit 16)

Let $L : K$ be a simple, purely inseparable extension.

Prove that the Galois group $\Gamma(L : K)$ is trivial. [10]

[END OF QUESTION PAPER]

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