Loughborough University

COMPUTATIONAL FLUID DYNAMICS 1
(08TTC001)

September 2008 | 2 Hours

Answer **THREE** questions.

Only University-approved calculators are permitted

1. a) Write brief notes on the following:

   i) The generalised, curvilinear co-ordinates system. [3 marks]

   ii) The basis of the multi-grid method used to achieve the best convergence acceleration for iterative techniques. [5 marks]

   b) Reduce the full form of the following scalar conservation equation to the version applicable to steady one-dimensional constant velocity flow in the x-direction with constant fluid properties.

$$\frac{\partial \rho\varphi}{\partial t}+\frac{\partial}{\partial x}(\rho u\varphi)+\frac{\partial}{\partial y}(\rho v\varphi)+\frac{\partial}{\partial z}(\rho w\varphi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma\frac{\partial\varphi}{\partial z}\right)$$

   Subject to the boundary conditions:

   $x = 0 \quad \varphi = 1$

   $x = L \quad \varphi = 0$

   Show that the exact solution is:

$$\varphi=\left(e^{Pe\frac{x}{L}}-e^{Pe}\right)/\left(1-e^{Pe}\right)$$

   where $Pe = \dfrac{\rho UL}{\Gamma}$ (Peclet number) [12 marks]

2. a) A one dimensional viscous flow is governed by the following equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\upsilon\frac{\partial^2 u}{\partial x^2}$$

Where u: the velocity component in x-direction
t ; time
ν: kinematic viscosity

i) Classify this equation as parabolic or hyperbolic.
ii) Draw the domain of dependence/region of influence diagram in (x,t) space.

[10 marks]

b) Derive the algebraic finite difference equation for the following transient heat conduction in one space dimension, x, with constant thermal diffusivity, α, using a first order forward differencing in time and a second order centred difference in space (i.e. FTCS):

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$$

Where T: temperature
t; time

[10 marks]

3. Consider the stationary diffusion equation

$$\frac{\partial^2 u}{\partial x^2}=-4 \text{ with } \begin{matrix} x=0 \ , \ u=0 \\ x=1 \ , \ u=0 \end{matrix}$$

i) Evaluate the exact solution. [4 marks]

ii) Derive the governing algebraic equations using second-order central differencing with Δx = 0.25. [2 marks]

iii) Decompose the coefficient matrix A into three matrices D, E, F, where:

D: the main diagonal of matrix A
E: the lower triangular part of matrix A.
F: the upper triangular part of matrix A. [4 marks]

iv) Use the following point Jacobian solution method and solve the equations for three iterations.

$$\underline{\underline{D}}\,\underline{u}^{n+1}=B-\left(\underline{\underline{E}}+\underline{\underline{F}}\right).\underline{u}^{n}$$

Where n is an iteration counter. [2 marks]

v) Examine the residual, R, for the three iterations in (iv) above using:

$$\underline{\underline{R}}^{n+1}=\underline{\underline{A}}\,.\,\underline{u}^{n+1}-\underline{B}$$

[8 marks]

4. Deduce, using finite-volume methods, the algebraic equation corresponding to the following steady-state, first-order pde:

$$\frac{\partial(\rho U \varphi)}{\partial x} + \frac{\partial(\rho V \varphi)}{\partial y} = 0$$

Assume the two velocity components are everywhere uniform $U = U_o$ and $V = V_o$ (both positive) and use the following interpolating practices for the cell face values of φ:

i) linear profile between mesh nodes

eg. $\varphi_{i-\frac{1}{2}} = \frac{1}{2}(\varphi_i + \varphi_{i-1})$

ii) first order upwind interpolation

eg. $\varphi_{i-\frac{1}{2}} = \varphi_{i-1}$

In each case (i)&(ii) express the algebraic equations in the form:

$$a_P \varphi_P = a_N \varphi_N + a_S \varphi_S + a_E \varphi_E + a_W \varphi_W + S$$

Under what condition is the coefficient matrix diagonally dominant for each of the approximations (i)&(ii)? [20 marks]

5. a) Briefly describe the meaning of the following with relevance to the solution of a finite difference equation:

i) Unstable solution
ii) Stable solution
iii) Discretization error [6 marks]

b) Derive the stability condition using **"Von-Neuman stability"** analysis for the following inviscid one dimensional linearised wave equation:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \qquad \text{(c = constant and positive)}$$

In your answer, use the following finite difference discretisation methods:

- forward in time
- first order backward approximation for the spatial gradient

[14 marks]

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