Loughborough University

# COMPUTATIONAL FLUID DYNAMICS - 1
(07TTC001)

January 2008 | 2 Hours

Answer **THREE** questions

Only University-approved calculators are permitted

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1. a) Use control volume analysis to drive the following Cartesian form of the continuity equation for a 2D unsteady flow:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

[8 marks]

b) Show how this may be transformed into the form shown below suitable for solution on a curvilinear, boundary conforming mesh $(\xi, \eta)$ :

$$\xi = f_1(x, y), \qquad \eta = f_2(x, y)$$

$$\frac{\partial \rho^*}{\partial t} + \frac{\partial f^*}{\partial \xi} + \frac{\partial g^*}{\partial \eta} = 0$$

***where*** $\rho^* = J\rho, \quad f^* = (\rho u)y_\eta - (\rho v)x_\eta, \quad g^* = (\rho v)x_\xi - (\rho u)y_\xi$

**where J is the Jacobian of the transformation**

$$J = x_\xi y_\eta - x_\eta y_\xi$$

$$\left(\textbf{assume :} \quad \xi_x\left(= \frac{\partial \xi}{\partial x}\right) = \frac{y_\eta}{J}, \xi_y = -\frac{x_\eta}{J}, \eta_x = -\frac{y_\xi}{J}, \eta_y = \frac{x_\xi}{J}\right)$$

[12 marks]

2. a) By considering the flux balance for a rectangular control volume of elemental size dx, dy, dz, derive the following conservation equation for a scalar quantity $\phi$ from first principles by the 'long hand' route described in the lectures; state any assumptions you make.

$$\frac{\partial \rho\phi}{\partial t}+\frac{\partial}{\partial x}(\rho u\phi)+\frac{\partial}{\partial y}(\rho v\phi)+\frac{\partial}{\partial z}(\rho w\phi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial \phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma\frac{\partial \phi}{\partial z}\right)$$

[16 marks]

b) Reduce the full form of the scalar conservation equation derived above to the version applicable to steady, 1D, and constant velocity flow in x-direction with constant fluid properties. [4 marks]

3. a) Express the following continuity and x-momentum equations in conservation tensor notation form for incompressible flows:

continuity:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

x-momentum:

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial}{\partial x}\left(\rho u^2\right)+\frac{\partial}{\partial y}(\rho uv)+\frac{\partial}{\partial z}(\rho uw)$$
$$=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\tau_{xx})+\frac{\partial}{\partial y}(\tau_{yx})+\frac{\partial}{\partial z}(\tau_{zx})$$

[10 marks]

b) Illustrate the application of the finite-volume method to Laplaces' equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

on a uniform rectangular Cartesian mesh, using an appropriate interpolation method. Assume flux is constant over cell surface. [10 marks]

4. Write brief notes on the following:

a) The advantages of finite – volume over finite – difference methods

b) The meaning of boundary – conforming co-ordinates and their benefits

c) The origin and cure for numerical diffusion

d) Point and line Gauss-Seidel iterative methods

[5 marks each]

5. Describe the type of pde appropriate to each of the problems listed below:

a)
- i) unsteady heat conduction
- ii) 2D steady boundary layer flow
- iii) 2D steady viscous transonic flow
- iv) steady inviscid supersonic flow

[8 marks]

b) Draw sketches and provide formulae (or explanations) which describe how a cell face dependent variable value $\left(\varphi_{i-\frac{1}{2}} \, say\right)$ may be interpolated from surrounding nodal values in 1D (x – direction) using the following approximations:

- i) First order upwind differencing
- ii) Second order upwind differencing
- iii) QUICK differencing

[12 marks]

**SALAH S IBRAHIM**