Loughborough University

COMPUTATIONAL FLUID DYNAMICS
(06TTC001)

January 2007 | 2 Hours

Answer **THREE** questions

Only University-approved calculators are permitted

All questions carry equal marks

1. Briefly describe the following:

   a. The benefits of a boundary conforming co-ordinate system and state two advantages of the derived equations for numerical solution.

   b. Transportive, Dissipative and Dispersive properties of differencing schemes for convective dominated problems.

   c. The difference between stable and converged numerical solution.

   d. The multi-grid method of enhancing convergence rate.

[5 marks each]

2. By considering the flux balance for a rectangular control volume of elemental size dx, dy, dz, derive the conservation equation for a scalar quantity ϕ per unit mass from first principles. State any assumptions you make. The derivation should allow for variable density (ρ) and variable diffusion co-efficient (Γ)

[20 marks]

3. Reduce the full form of the scalar conservation equation below, to the version applicable to steady one-dimensional constant velocity flow in the x-direction with constant fluid properties.

$$\frac{\partial \rho\phi}{\partial t}+\frac{\partial}{\partial x}(\rho u\phi)+\frac{\partial}{\partial y}(\rho v\phi)\frac{\partial}{\partial z}(\rho w\varphi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma\frac{\partial\phi}{\partial z}\right)$$

Subject to boundary conditions:

at $x = 0$, $\phi = 1$;
at $x = L$, $\phi = 0$

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Show that the exact solution is:

$$\phi = \left(e^{Pe\frac{x}{L}} - e^{Pe} \right) / \left(1 - e^{Pe}\right)$$

where $Pe = \dfrac{\rho U L}{\Gamma}$ *(Peclet number)*

sketch this solution for $Pe = 0, \ \pm 5, \ \pm 500$ and discuss the implication of the shapes of profiles obtained.

[20 marks]

4. Non-linear 1D wave-motion in a fluid is governed by the following equations:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x}$$

$$\rho a^2 \frac{\partial u}{\partial x} + \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} = 0$$

where 'a' is the speed of sound.

a. Classify this system of equations as hyperbolic or elliptic [14 marks]

b. Determine the characteristic directions in (x, t) space [2 marks]

c. Draw the domain of dependence / region of influence diagram [4 marks]

5. a. Apply von Neuman stability analysis to the following 1-D transport equation:

$$\frac{\partial \varphi}{\partial t} + u \frac{\partial \varphi}{\partial x} = \alpha \frac{\partial^2 \varphi}{\partial x^2} \quad \textit{(u, } \alpha \textit{ positive and constant)}$$

discretised via the Forward Time, Central Space (FTCS) approximation; show that the amplification factor G is given by :

$$G = 1 - 2s(1 - \cos\theta) - iC \sin\theta$$

where $s = \alpha \Delta t / \Delta x^2$, $C = u \Delta t / \Delta x$, and deduce the stability conditions for the scheme.

[15 marks]

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b. Explain why this method might be restrictive for obtaining steady-state solutions on non-uniform meshes. [5 marks]

SALAH IBRAHIM