Loughborough University

COMPUTATIONAL FLUID DYNAMICS
(05TTC001)

January 2006 | 2 Hours

Answer **THREE** questions

Use of calculators is permitted but all memory and stored programs must be cleared.

All questions carry equal marks

1. a) Give a brief description of the following properties of finite difference schemes:

- stability [2 marks]
- consistency [2 marks]
- numerical accuracy [2 marks]
- boundedness [2 marks]
- conservation [2 marks]

b) For the unsteady two-dimensional scalar transport equation, centred spatial differencing, and implicit time differencing, derive the set of algebraic finite difference equations. Assume constant density, constant diffusion coefficient, Γ, and a uniform structured grid of spacing Δx and Δy [10 marks]

2. a) Use the ideas of a domain of dependence/ region of influence to explain the difference between parabolic, hyperbolic and elliptic p.d.e's. Give an example of a flow problem governed by each of these equation types.

[10 marks]

b) Find the characteristic directions for the following equations governing 2D potential (i.e. inviscid irrotational) flow:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0$$

continued......

Is this system elliptic, hyperbolic or parabolic? What implication does this have for boundary condition specification? [10 marks]

3. a) Consider the 1D unsteady scalar convection equation:

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0 \quad ; \quad u > 0 \text{ and constant}$$

Use forward time, backward space differencing to produce a finite difference analogue of this equation. [3 marks]

b) Apply von-Neumann stability analysis to show that the amplification factor G is given by:

$$G = (1 - C) + C(\cos\theta - i\sin\theta), \qquad C = u\frac{Dt}{Dx}$$

and deduce the stability constraint of the method in terms of the Courant number C. [14 marks]

c) Explain why the above condition might be restrictive for obtaining a steady - state solution on non-uniform meshes. [3 marks]

4. Briefly describe the following: [5 marks each]

a) The difference between explicit and implicit methods and their relative advantages and disadvantages.

b) The CFL condition

c) Direct and iterative methods for the solution of sets of linear algebraic equations; give two examples of the latter method, one point based and one line-based.

d) The difference between finite-difference and finite-volume discretisation methods

5. Show how, by making appropriate substitutions, the second order equation:

$$\frac{\partial^2 u}{\partial t^2} - \beta \frac{\partial^2 u}{\partial x^2} + u = 0$$

may be converted into the equivalent first order system:

$$\frac{\partial u}{\partial t} - w = 0$$

$$\frac{\partial v}{\partial t} - \frac{\partial w}{\partial x} = 0$$

$$\frac{\partial w}{\partial t} - \beta \frac{\partial v}{\partial x} = -u$$

and hence show that this equation is hyperbolic if $\beta > 0$ with characteristics $\pm \beta^{1/2}$.

[20 marks]

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