UNIVERSITY OF LONDON

279 005a ZB
990 005a ZB
996 D05a ZB

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 1 (half unit)

Thursday, 11 May 2006 : 10.00am to 12.00noon

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from Section A (60 marks in total) and **TWO** from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided. If used, it must be fastened securely inside the answer book.

Calculators may **not** be used for this paper.

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SECTION A

Answer all **seven** questions from this section (60 marks in total).

1. The demand equation for a good is given by $p = q^2 + 6q + 24$. Sketch the demand curve for $q \geq 0$. If the supply equation is $p = -q^2 - 8q + 180$, determine the equilibrium price and quantity.

2. A monopoly experiences a demand q for its product that is related to its price by $qp^2 = 64$. The cost to the company of supplying q units is $2q^2$. Determine the price and quantity when the firm maximises its profit.

3. Use a matrix method to determine the numbers x, y, z which satisfy the following three equations:

$$x - 2y + 5z = 5, \quad 2x + y - z = 4, \quad x + 3y + 2z = 7.$$

4. Determine the integral

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\, dx.$$

5. Two functions $V(x, y)$ and $U(x, y)$ are connected by the equation

$$V(x, y) = U(x, y)e^{-ax-by}$$

where a and b are constants. Find

$$\frac{\partial V}{\partial x}, \quad \frac{\partial V}{\partial y}, \quad \frac{\partial^2 V}{\partial x^2}$$

in terms of U and its partial derivatives.

Suppose that V satisfies

$$\frac{\partial V}{\partial y} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial x} - 2V.$$

Let

$$a = \frac{1}{2}, \quad b = \frac{9}{4}.$$

Show that the function U then satisfies the equation

$$\frac{\partial U}{\partial y} = \frac{\partial^2 U}{\partial x^2}.$$

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6. The function $f(x, y)$ is given, for $x, y \neq 0$, by

$$f(x, y) = \frac{4}{x} - \frac{2x}{y} + y.$$

Show that f has one critical (or stationary) point and determine what type of critical point this is.

7. Find the values of x and y that will maximise the function $(x+1)^2(y+1)^3$ subject to the constraint $x + y = 13$.

SECTION B

Answer **two** questions from this section (20 marks each).

8.(a) Three goods are sold in the same market. If their prices are x_1, x_2, x_3, then the demand quantities y_1, y_2, y_3, and the supply quantities z_1, z_2, z_3 are given by the following equations:

$$\begin{aligned} y_1 &= 50 - x_1 + x_2 + x_3 \\ z_1 &= 2x_2 + x_3 - 20 \\ y_2 &= x_1 - x_2 - 2x_3 + 264 \\ z_2 &= 2x_1 + 4x_2 + 5x_3 - 30 \\ y_3 &= 2x_1 + 2x_2 - 2x_3 + 4 \\ z_3 &= 8x_1 + 2x_3 - 40. \end{aligned}$$

Non-negative numbers x_1^*, x_2^*, x_3^* are said to be equilibrium prices if, when the prices are $x_1 = x_1^*, x_2 = x_2^*$ and $x_3 = x_3^*$, then the supply and demand quantities for each good are equal; that is, $y_1 = z_1$, $y_2 = z_2$, and $y_3 = z_3$. **Using matrix methods**, find the equilibrium prices.

(b) Determine the following integrals:

$$\int_0^{\pi/4} \frac{1}{(2\tan x + 1)\cos^2 x}\, dx,$$

$$\int (x+1)^2 \ln x \, dx$$

9.(a) A firm's marginal cost function is $(q+1)/(q^2+5q+6)$. If production is increased from $q=1$ to $q=2$, find the increase in cost.

(b) A depreciating asset whose initial value is A has a value at time t equal to $A(1-st)$, where $s>0$ is its rate of depreciation. If the asset is sold at time t then the 'present value' of the money raised by the sale is given by

$$P(t) = Ae^{-rt}(1-st),$$

where $r>0$ is the bank interest rate. Find the value of t at which $P(t)=0$ and find also the value of t when $P(t)$ has a critical (or stationary) point. Sketch the graph of $P(t)$.

(c) Find the first-order partial derivatives, using the product rule, of the function

$$f(x,y) = (x^2+y^2-6)(xy+5).$$

Hence determine the critical (or stationary) points of the function. [You will find it helpful to consider the sum of the two first-order partial derivatives.]

10.(a) An ex-student donates a sum of money to his former school in order to provide an annual prize of $\$P$, to be awarded N times, starting one year from the date when he makes the donation. The school plans to invest the money in a bank account paying interest annually at a rate of 5%. Find an expression, in as simple a form as possible, for the minimum donation that the ex-student will have to make to the school. Justify your answer fully, showing all steps in your reasoning and calculations.

(b) A firm has production function

$$q(k,l) = \left(k^{\beta} + 2l^{\beta}\right)^{1/\beta}$$

where $0<\beta<1$. Here, k denotes units of capital and l denotes units of labour. Each unit of capital costs \$1 and each unit of labour costs \$1. Find an expression, in terms of β and M, and in as simple a form as possible, for the maximum amount of its good that the firm can produce if it spends no more than an amount $\$M$ on capital and labour.

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11.(a) Suppose that

$$f(x, y) = \sqrt{x + y + \sqrt{x^2 + y^2}}.$$

Determine $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ and show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{1}{2}f(x, y).$$

(b) A monopolist produces two goods, X and Y and the inverse demand functions for these are given by

$$p_X = 2 - 2x, \quad p_Y = 4 - 2y$$

where p_X and p_Y are the prices of X and Y, respectively and x, y denote the production levels of X and Y, respectively. Suppose the firm has joint total cost function

$$TC = x^2 + kxy + y^2$$

where k is a positive constant and $k < 3$.

Show that the monopolist's profit function will have a maximum value for some positive values of x and y. Determine these values in terms of k.

END OF PAPER