This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON 279 0076 ZB

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Management Mathematics

Monday, 12 June 2006 : 10.00am to 1.00pm

Candidates should answer **FIVE** of the following **EIGHT** questions. All questions carry equal marks.

Graph paper is provided. If used, it must be fastened securely inside the answer book.

New Cambridge Statistical Tables (second edition) are provided.

A hand held calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

PLEASE TURN OVER

UL06/94

1. A company undertakes a survey of its 110 adult employees and discovers that there are:

10 unmarried men without degrees,
50 married employees,
60 employees with degrees,
30 unmarried women without degrees
20 women with degrees,
15 married women.

(a) Draw a Venn diagram (with *W, D, M* denoting 'women', 'has degree' and 'married' respectively) in order to determine the maximum and minimum number of women who are married and have a degree? **(10 marks)**

(b) On the assumption that the number of married women with degrees takes its minimum value, construct a fully annotated Venn diagram (with *W, D, M* denoting 'women', 'has degree' and 'married' respectively) to show the order of each subset. **(4 marks)**

(c) Making use of the diagram in (b) above, describe each of the following subsets in words and state their order:

i. $(W \cup D)^c$
ii. $(W \cap D^c \cap M^c)$
iii. $M \cup (W \cap D^c)$

(6 marks)

2. A model of industrial output *P(t)* for a country at time *t* is given by

$$a\frac{d^2P}{dt^2} + b\frac{dP}{dt} - cP = 0$$

where *a, b* and *c* are positive constants and, when $t = 0$, $P = 100$ and $dP/dt = 12$

(a) Given $a = 0.1$, $b = 2$ and $c = 2.1$
i. Derive the solution for *P(t)* as a function of *t*. **(8 marks)**
ii. Produce a graph of *P(t)* against *t* and determine when *P(t)* first reaches 200. **(8 marks)**

(b) Suppose the constants *a, b* and *c* remain as above and *P* is still 100 when $t = 0$, what range of values for *dP/dt* at $t = 0$ will lead to a monotonically decreasing P? **(4 marks)**

PLEASE TURN OVER

3. An American car manufacturer produces three different models of car. Details of selling price (x US$1000) per car and annual sales revenue (x US$1,000,000) are given below for the years 2003 to 2005.

	2003		2004		2005	
Model	Price	Revenue	Price	Revenue	Price	Revenue
Basic	25.0	500	26.0	650	25.0	750
Family	30.0	300	32.0	400	35.0	350
Executive	55.0	330	60.0	330	60.0	420

(a) Produce the following index series for the manufacturer:

i. A Laspeyres weighted price index. **(5 marks)**

ii. A Paasche weighted production quantity index. **(5 marks)**

iii. An unlinked chain-based price index for the Basic model. **(2 marks)**

(b) A measure of inflation in the USA is given by the following index series (2000=100):

Year	2003	2004	2005
Retail Price Index(2000=100)	120.0	122.0	130.0

i. Deflate the price index produced in (a)i. above and comment upon your answers. **(5 marks)**

ii. Briefly discuss the shortcomings of using this deflated series as a measure of whether the car manufacturer has been increasing its car prices at a reasonable rate between 2003 and 2005. **(3 marks)**

4. Historically, the annual rainfall in a particular wine growing area has varied very little away from 80cms. However it has become more variable over the past few years as follows:

Year	Annual Rainfall (cms)
2000	70.0
2001	90.5
2002	80.3
2003	65.2
2004	45.7
2005	71.1

(a) Using exponential smoothing with a smoothing constant of $\alpha = 0.20$, develop forecasts for 2000 through to 2006. What is the Root Mean Square Error of your forecast between 2001 and 2005? **(8 marks)**

(b) Develop a simple three year moving average forecast for the same data and compare the accuracy of your two forecasting models. **(6 marks)**

(c) Discuss how you might improve on the forecasting models suggested above. **(6 marks)**

5. A partitioning (non-hierarchical clustering) algorithm has identified the centres of four clusters, defined by three variables A, B and C, given below:

Cluster	Variable A	B	C
1	6	3	7
2	5	4	9
3	7	6	5
4	1	3	1

(a) A new observation (4,6,2) occurs. If the distance between two points (a_1,b_1,c_1) and (a_2,b_2,c_2) is $\sqrt{(a_1-a_2)^2+(b_1-b_2)^2+(c_1-c_2)^2}$, to which cluster should the new observation be allocated and why? **(4 marks)**

(b) If the distance between the two points is $|(a_1+b_1+c_1)-(a_2+b_2+c_2)|$ to which cluster should the new observation be allocated and why? **(4 marks)**

(c) If the distance between two **clusters** (a_1,b_1,c_1) and (a_2,b_2,c_2) is $|a_1-a_2|+|b_1-b_2|+|c_1-c_2|$ determine the distances between the four given clusters and use single linkage hierarchical clustering to reduce the number of clusters from 4 down to 3 and then down to 2. Produce an appropriate dendogram for this process. **(12 marks)**

6. The following inter-industrial flow table shows the amount of A, B and C's output that is required as input for A, B and C's production. The total gross production is 356 of A, 2400 of B and 420 of C. Excess production is used to satisfy external demand.

Input to	Output from A	B	C
A	267	178	178
B	0	1920	192
C	84	252	0

(a) Determine the satisfied external demand for each product. **(2 marks)**

(b) Write down the technological matrix. **(4 marks)**

(c) Assuming the same technological matrix applies (and using it!), calculate the new gross productions required in each industry to meet external demands of 10 of A, 100 of B and 30 of C. **(10 marks)**

(d) What external demand could be satisfied by a gross production of 500 of A, 3000 of B and 500 of A? **(4 marks)**

PLEASE TURN OVER

7. A machine is inspected at equally spaced time intervals and after each inspection it is classified into 4 states, 1,2,3,4 or 5. A machine in state 1 is unused, a machine in state 5 is worn out or broken irreparably. States 2 to 4 represent increasing amount of usage. Sometimes a machine in state 5 is replaced with a 'second-hand' one. The times of inspection are denoted by $t = 0,1,2,\ldots$
If X_t denotes the observed state of the machine at time t then $\{X_t\}$ is a Markov Chain with transition matrix P given by

$$P = \begin{pmatrix} \frac{1}{10} & \frac{4}{5} & 0 & 0 & \frac{1}{10} \\ 0 & \frac{1}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} & \frac{4}{5} & 0 \\ 0 & 0 & 0 & \frac{3}{5} & \frac{2}{5} \\ \frac{4}{5} & 0 & \frac{1}{5} & 0 & 0 \end{pmatrix}$$

(a) Evaluate the equilibrium state probabilities. **(12 marks)**

(b) Draw a network to illustrate the above transition matrix. **(3 marks)**

(c) What is the average time period for a machine in state 4 to reach state 5 ? **(5 marks)**

8.(a) What are the main assumptions involved in using ordinary least squares regression for estimation purpose? **(8 marks)**

(b) What are the uses of a 'box and whisker' (sometimes just referred to as a 'box') diagram? **(2 marks)**

(c) Construct a 'box and whisker' diagram for each of the following data sets (A and B) and compare them:

Observation Number	1	2	3	4	5	6	7	8	9	10
Value for Set A	2	72	25	123	194	156	269	71	50	57
Values for Set B	80	21	47	56	50	60	42	71	48	61
Observation Number	11	12	13	14	15	16	17	18	19	
Value for Set A	67	8	65	81	68	61	62	56	7	
Value for Set B	102	56	71	50	29	54	61	42	57	

(10 marks)

END OF PAPER