PAPER CODE NO.
**MATH724**

THE UNIVERSITY
*of* LIVERPOOL

# MAY 2006 EXAMINATIONS

Bachelor of Science : Year 3
Master of Mathematics : Year 3
Master of Mathematics : Year 4

## INTRODUCTION TO THE METHODS OF APPLIED MATHEMATICS

TIME ALLOWED : Two Hours and a Half

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INSTRUCTIONS TO CANDIDATES

Full marks can be obtained for complete answers to FIVE questions. Only the best FIVE answers will be counted. Marks for parts of questions may be subject to small adjustments.

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**1.** (i) Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{3xy}{(x^2-1)}$$

putting your answer in the form $y = f(x)$.

**[6 marks]**

(ii) Solve the initial value problem

$$x^3\frac{dy}{dx} - xy = 2e^{-\frac{1}{x}}$$

for $y(x)$ where $y(1) = 0$.

**[6 marks]**

(iii) Solve the initial value problem

$$\frac{dy}{dx} = \frac{x\cos(y^2)}{[x^2 y\sin(y^2) + 2y^3]}$$

where $y(1) = 0$.

**[8 marks]**

**2.** Solve the differential equation

$$y'' - 6y' + 9y = 2e^{ax}$$

for the case $a \neq 3$ with the initial condition $y(0) = 0$ and $y'(0) = 3$.

**[11 marks]**

For the case $a = 3$, determine $y(x)$ subject to the same boundary conditions.

**[9 marks]**

**3.** Find the general solution of the system of differential equations

$$\frac{d^2x}{dt^2} = 3x - y + 2e^{2t}$$
$$\frac{d^2y}{dt^2} = -6x + 4y - e^{2t}$$

when $x(0) = 0$, $y(0) = 0$, $\dot{x}(0) = 0$ and $\dot{y}(0) = 0$.

**[20 marks]**

**4.** Solve the differential equation for $y(x)$

$$x^2y'' - 5xy' + 9y = 4x^4 + 3x\ln x$$

with the initial conditions $y(1) = 2$ and $y'(1) = 5$.

**[20 marks]**

**5.** The function $u(x, y)$ satisfies the first order partial differential equation

$$x\frac{\partial u}{\partial x} + 2x^2\frac{\partial u}{\partial y} = u + y^2$$

subject to the boundary conditions $u(1, y) = \sinh(y)$.
Determine the function $u(x, y)$.

**[20 marks]**

**6.** The function $f(x)$ has period $2\pi$ and also satisfies

$$f(x) = \begin{cases} 2 & 0 < x < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} < x < \pi \end{cases}.$$

(i) If $f(x)$ is even, sketch the graph of $f(x)$ for $-2\pi < x < 2\pi$ and find its Fourier series.

**[10 marks]**

(ii) If $f(x)$ is odd, sketch the graph of $f(x)$ for $-2\pi < x < 2\pi$ and find its Fourier series.

**[10 marks]**

**7.** By seeking a solution of the form $u(x, y) = F(x)G(y)$, show that

$$u(x, y) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right)\left[A_n \cosh\left(\frac{n\pi y}{L}\right) + B_n \sinh\left(\frac{n\pi y}{L}\right)\right]$$

solves Laplace's equation in the square $0 \le x, y \le L$ with boundary conditions $u(0, y) = u(L, y) = 0$.
Find the particular solution for which $u(x, 0) = 0$ and $u(x, L) = x(L - x)$.

**[20 marks]**