Marks will be awarded to the best five answers.
All questions carry equal marks.

1. This is a game for two players, I and II. The game starts with each player staking, in the kitty, £2.00. The referee then gives each player a set of two cards, one red and one blue. Each player chooses a card from their set and gives it to the referee, who is the only person to see both cards. The referee keeps these cards. If both cards are red, he stops the game and declares II the winner. Otherwise a green card is given to each of the players. I and II now stake a further £2.00 . They both choose simultaneously a card from their set. If the cards are the same II wins the kitty and if different I wins it.

Draw the extensive form of this game. *(10 marks)*

What are the pure strategies of player I? *(5 marks)*

Give an example of a behavioural strategy, which includes every possible action, for both I and II. *(5 marks)*

2. Explain the terms *subgame* and *subgame perfect equilibrium.* *(2 marks)*

In the game $G[A : (m_0, n_0)]$, two players alternately take discs from one of two piles on a table. The discs are identical and we designate the numbers in the piles as (m,n), where $m \geq n$. A is the name of the player who goes first and (m_0, n_0) are the initial values of (m, n). Players cannot pass; the player who moves last loses. A move for either player is, if $m > n$, either to remove either one or two discs from the larger pile, and if $m > n + 1$ then, in addition, one disc can be moved from the larger pile to the smaller pile. However if $m = n$ then just one disc, or one from each pile, can be removed.

Draw the game tree for the games $G[A : (1, 1)]$ and $G[A : (2, 1)]$, taking B to be the other player. Find out who can ensure that they can win these games. *(6 marks)*

Hence draw game-trees for $G[A : (2, 2)]$ and $G[A : (3, 1)]$ in terms of $G[B : (1, 1)], G[B : (2, 1)]$ and $G[B : (2, 2)]$. Determine who can ensure a win in these games. Similarly draw trees for $G[A : (3, 2)], G[A : (3, 3)], G[A : (4, 3)]$ and $G[A : (4, 4)]$ and find who can ensure a win in each game.

(12 marks)

3. Tesburys and Coopda are two supermarkets in the town of Birkenpool. Coopda has a large store and has most of the market share in the town whereas Tesburys store is small, with a much smaller market share. The management of Tesburys considers whether to invest and make their store larger, which will take market share from Coopda and other, smaller, stores. At present the profits of the stores are (2;200) in units of 1,000 per week to Tesburys and Coopda respectively. Tesburys are not sure about the reaction of Coopda: they may accept Tesburys enlargement, whereby the profits would become (120; 120) units per week, or they may reduce their prices and try to make Tesburys lose 40 units per week. This action would cost Coopda and their profits would drop to 50 units a week, but they may dislike Tesburys so much that to see Tesburys lose money would be worth an extra 100 units a week. Draw the extensive forms for the two possible games that may be played. *(4 marks)*)

Tesburys are not sure what type of management Coopda has, and ascribes a probability p to the management being friendly. Describe the above situation in terms of *beliefs, types* and *scripts* of a game of incomplete information, and draw the extensive form of the game of imperfect information that Harsanyi's theory suggests. Solve this game and find out how Tesburys decisions depend on p. *(16 marks)*

4. Define the terms *efficiency, imputation, core, stable set* for a cooperative n-person game. *(5 marks)*

Four sellers I, II, III and IV have identical mobile phones, which they are willing to sell at £25, £35, £40 and £55 respectively. A, B, C and D are four buyers and are respectively willing to spend £27, £32, £41 and £57 to buy such a mobile phone. Draw the demand and supply functions for this two-sided market and deduce the number of phones sold and the limits on the (single) price at which they are sold. *(7 marks)*

Consider this two-sided market as a 8-person cooperative game and find the values $v(I,A), v(I,B), v(I,C), v(I,D), v(II,C), v(II,D), v(III,A), v(III,D)$, $v(IV,A), v(I,A,B), v(I,II,C,D)$ and $v(I,II,III,IV,A,B,C,D)$ of the characteristic function, and any others that you think necessary. Hence find the core and the limits on the (single) price of phones in this market. Compare your results with those obtained above. *(8 marks)*

5. Define a *simple voting game* in terms of the winning subsets of a set N. What is a weighted voting game? Give an example of a simple voting game which is not a weighted voting game. (*5 marks*)

A weighted voting game is written $\{m \mid 6, 4, 2, 2, 1\}$ with players A, B, C, D and E. Find the minimum-winning player-sets for $m = 9$ and 10, and show that C, D and E are interchangeable when $m = 9$, but E is a dummy player when $m = 10$. (*4 marks*)

When $m = 9$, find the player-sets in which A, B, and C are separately pivotal. Calculate the Shapley-Shubik and the normalised Banzhaf indices for this game. (*11 marks*)

6. Define an Evolutionarily Stable Strategy for a symmetric game with payoff matrix A. Why are all such strategies Nash equilibria, but all Nash equilibria not always E.S.S.s? (*5 marks*)

In a simple model of sperm competition for a particular mammalian species, there are a large number of adult males and females, with more females than males. Each female mates twice, the second mating being less successful in producing offspring by a factor r. Males are equally likely to mate first or second, ejaculating s_1 and s_2 respectively. From a first mating the number of offspring which survive to maturity is $Gs_1/(s_1 + rs_2)$and from a second it is $Grs_2/(s_1 + rs_2)$, where G is a constant. Let n be the number of matings a typical male has (n, s_1 and s_2 can be taken to be continuouus variables). Write down an 'energy' constraint equation involving n, s_1, s_2, and show how the consideration of mutations from the E.S.S. determine the E.S.S. values $s{*}_1, s{*}_2$ of the sperm numbers.

Show also that the values $s{*}_1, s{*}_2$ are equal. (*15 marks*)

7. (*a*) Discuss two of the following topics in game theory:
(*i*) *signalling*:
(*ii*) *war of attrition.*
(*iii*) *coalitions, including the ideas of partitions, superplayers and the preference rule.*

Mathematical reasoning is preferred. In (*i*), provide at least one extensive form of a signalling game. (*16 marks*)

(*b*) Explain the terms in and justify the replicator equation

$$\frac{dp_i}{dt} = p_i(f_i(p) - \bar{f}).$$

(*4 marks*)