1. In a game for two players, A and B stake themselves in by putting £1 into the kitty. Player A takes unseen a card from a hat; initially there are 4 cards in the hat, one with "2" on it and 3 with "0" written on them. A looks at the card but does not show it to B.

Player A then announces whether he "raises" or "sticks" ; (in this game, a "Raise" means a player puts £7 into the kitty and gains a point; a "Stick" means that a player puts £1 into the kitty). Next B takes unseen a card from those remaining in the hat and then announces whether he "raises", or "folds"; (a "fold" means that B concedes the game giving all the kitty to A).

A player gets a total number of points that is the number on the card he gets plus any point he gets from raising. The player with the larger number of points wins the kitty and the kitty is shared if the points are equal.

Draw the game-tree. What are the pure strategies of the players? Calculate, showing your working, the element of the payoff matrices of the players when they always raise. [20 marks]

2(a). A and B have VNM utility functions U^A, U^B respectively. The values for the sure prospects $s_i, i = 1, 2, 3, 4$ are as follows:

$$\begin{array}{ccccc} & s_1 & s_2 & s_3 & s_4 \\ U^A & 4 & 3 & -5 & 1 \\ U^B & -2 & 2 & -4 & -6 \end{array}$$

Let s_o, s_w denote the "best" and "worst" prospects respectively for A. Let $s(p)$ denote the risky prospect, $[ps_o, (1-p)s_w]$, for A where p is a probability. Find separately for each sure prospect a value of p for a risky prospect so that A feels indifferent between that risky prospect and the sure prospect. Do the same for B. [10 marks]

(b) Eliminate the dominated, complete rows and complete columns of the payoff matrices for A, the row player, and B, the column player, respectively in the following two-person game:

$$\begin{pmatrix} -2 & -3 & 1 \\ 1 & -9 & 0 \\ -2 & -4 & 0 \end{pmatrix}, \begin{pmatrix} -4 & -3 & -9 \\ -4 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$$

Show that the remaining payoff matrices are for a "Prisoners' Dilemma"-type game, explaining carefully what you mean by such a game.

[10 marks]

3(a). In a bi-matrix game, the payoff matrices for A & B are

$$\begin{pmatrix} (1,1) & (-1,-2) \\ (-2,-2) & (0,0) \end{pmatrix}$$

Use the swastika method, or otherwise, find the Nash equilibria of this game.

[10 marks]

(b) Assume the minimax theorem in a zero-sum game. Suppose the payoff matrix in such a game for A is U_{ij} when player A and player B play strategies i and j respectively, and that the Nash strategies are $\mathbf{p}^* \equiv (p_1^*, p_2^*, \cdots, p_m^*)$ and $\mathbf{q}^* \equiv (q_1^*, \cdots, q_n^*)$. Suppose that the value of the game is v. Prove that if $\sum_j U_{ij} q_j^* < v$ then $p_i^* = 0$. [10 marks]

4. The following are pay-offs for player I in three zero-sum games:

$$(a) \begin{pmatrix} 1 & -3 \\ -2 & 2 \\ -1 & 1 \end{pmatrix} \quad (b) \begin{pmatrix} -2 & 3 & -3 \\ 0 & 2 & 1 \\ -1 & -2 & 2 \end{pmatrix} \quad (c) \begin{pmatrix} 4 & 1 & 3 \\ 0 & 8 & 2 \\ 1 & 5 & 4 \end{pmatrix}$$

Find the maximin solutions and the values of the games. [6, 3, 11 marks]

5(a). A two-player cooperative game has bi-matrix:

$$\begin{pmatrix} (4,1) & (5,3) & (0,5) \\ (0,1) & (8,0) & (4,2) \end{pmatrix}$$

Draw the attainment set for this game, indicating the pareto-Optimal set. Calculate the maximin values for this game and show that they are $(2,2)$. Calculate the maximin bargaining payoffs. [10 marks]

(b) The last strategy for the column player becomes unavailable, i.e. the bi-matrix of payoffs consists of the first two columns. Find the threat bargaining solution. [10 marks]

6. Explain the superadditivity property of the coalitional (sometimes called the characteristic) function of an n-person cooperative TU game. What is a dummy in such a game?

Three students, Andrew, Bertie and Caroline, want a particular DVD that other students managed to get. They have £12, £6 and £10, respectively that they are willing to spend on such an item. Other students, Deirdre, Edie and Freddie, have got one DVD each and are willing to part with theirs for at least £13, £7 and £9 respectively. They all have to meet together to agree on a common price for a DVD. Argue that there are two dummy players in this situation.

Calculate the coalitional function and derive the core solutions. What is the meaning of the imputation of a core solution? Suppose Caroline has to spend all her money: What is the price of the DVD and what money do the others end up with? [20 marks]

7. Abdul and Benjamin are traders in sheep and goats at the Jerusalem market. One day Abdul comes to market with 50 sheep and 10 goats, and Benjamin comes with 30 sheep and 70 goats.

Abdul's and Benjamin's preferences for sheep and goats are represented respectively by the utility functions:

$$U_A(s,g) = (s+15)(2g+10), \quad U_B(s',g') = (s'+10)(g'+50),$$

where s, s' and g, g' are their numbers of sheep and goats.

Draw the Edgeworth box and roughly sketch a few indifference curves for both of them. Show that the Pareto-optimal set for trading without money is the line $7g - 9s = 100$. What are the end-points of the contract curve (show your working). [20 marks]