Useful formulae

Single group, n independent observations $x_1, \ldots, x_n$ from a population with mean μ and variance σ^2:

- sample mean $\hat{\mu} = \bar{x} = \sum x_i / n$
- sample variance $\hat{\sigma}^2 = s^2 = \sum (x_i - \bar{x})^2 / (n-1)$ or $s^2 = \frac{n}{n-1}\left[\frac{\sum x_i^{\,2}}{n} - \bar{x}^2\right]$
- standard error of the sample mean is $\sigma / \sqrt{n}$

Two groups, n independent observations $x_1, \ldots, x_n$ from a population with mean μ_1 and m independent observations $y_1, \ldots, y_m$ from a population with mean μ_2 (common variance σ^2):

- standard error of mean difference is $\sigma\sqrt{(1/n) + (1/m)}$

1. A doctor is interested in the effect of a new treatment on the level of serum thyroxine in the blood of infants who have hypothyroidism (particularly low levels of serum thyroxine). To examine whether there is a difference between the new and the standard treatments she gives both these treatments in some random order to each of 15 infants. The levels of serum thyroxine in the blood for each infant on both treatments are given below, in nmol/l.

Infant	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
New Treatment	49	58	59	80	37	45	63	86	60	65	75	71	39	52	45
Standard Treatment	47	57	57	81	32	45	59	83	54	64	71	74	36	53	45

(a) Calculate the differences between treatments for each patient and display the data in a stem-and-leaf plot. Comment on the general shape of the distribution.

[7 marks]

(b) Perform a test of the hypothesis that there is no difference between the new and standard treatment. State the assumptions behind your test and comment, with reasoning, on whether you feel the assumptions are satisfied. Interpret your results.

[9 marks]

(c) The doctor tells you that to be clinically worthwhile, the new treatment would need to increase the serum thyroxine level by at least 2 nmol/l. Making further calculations to backup your opinion, would you say the results are clinically worthwhile?

[4 marks]

2. The following length data (y, in centimetres) were collected from a random sample of boys aged between 18 – 30 months (x) in Kalama, Egypt.

	Boy	1	2	3	4	5	6	7	8	9	10	11	12
Length (y)		76.1	77.0	78.1	78.2	78.8	79.7	79.9	81.1	81.2	81.8	82.8	83.5
Age (x)		18	19	20	21	22	23	24	25	26	27	28	29

$$\sum y = 958.2 \quad \sum x = 282 \quad \sum xy = 22608.5 \quad \sum x^2 = 6770 \quad SSE = 0.55$$

(a) The relationship between length and age can be modelled as

$$y_i = \alpha + \beta x_i + e_i \qquad (i = 1, 2, \ldots, 12)$$

where the e_i are independent Normal random variables each with mean zero and variance σ^2.

Show that the least squares estimates of α and β are given by

$$\hat{\beta} = \frac{\sum_i x_i y_i - n\bar{x}\,\bar{y}}{\sum_i x_i^2 - n\bar{x}^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\,\bar{x}$$

where $\bar{x}$ denotes mean age, $\bar{y}$ denotes mean length and n is the number of boys in the sample. [8 marks]

(b) Calculate $\hat{\alpha}$ and $\hat{\beta}$ for these data and interpret your results. [6 marks]

(c) A further boy is aged 21 months. What can be said about his predicted length?

[6 marks]

[Hint: a 95% prediction interval for a new response y_0 when $x = x_0$ is given by

$$\hat{\alpha} + \hat{\beta} x_0 \pm t_{n-2}(0.025)\, s\sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} \text{ where } s^2 = \frac{SSE}{n-2} \text{ and } S_{XX} = \sum (x_i - \bar{x})^2 .]$$

3. In a particular investigation the *temperature* in °C in each of two separate groups of children was measured and the data collected are given below:

Group 1 36.2, 36.5, 36.7, 36.8, 36.9, 37.0, 37.1, 37.3, 37.3, 37.4, 37.6, 37.7, 37.8, 37.9, 37.9, 38.0, 38.1, 38.2, 38.2, 38.3

Group 2 36.0, 36.1, 36.3, 36.5, 36.6, 36.7, 36.7, 36.8, 36.9, 37.0, 37.0, 37.2, 37.2, 37.2, 37.3, 37.4, 37.5, 37.9, 38.0, 38.3

(a) Define the *power* of a hypothesis test. [2 marks]

(b) The sample means for groups 1 and 2 are 37.45 and 37.03 respectively. A formal test of the equality of variances of the two groups was not rejected. The pooled sample variance, s_p^2, is calculated to be 0.38. Test the null hypothesis that there is no difference in temperature between the two groups. [6 marks]

(c) Fever is defined as a temperature higher than or equal to 37.8°C. Using the data above, complete the table below in terms of the number of patients in each group who did and did not have a fever.

	Fever	**No fever**	**Total**
Group 1			
Group 2			
Total			

Hence test the hypothesis that there is no association between group and fever.

[9 marks]

(d) Comment on any differences between the conclusions of the hypothesis tests in (b) and (c). If the conclusions differ, can you suggest why?

[3 marks]

4. Wild bears were anaesthetised, and their bodies were measured and weighed. One goal of the study was to predict the weight of a bear from other measurements, since in the forest it would be easier, for example, to measure the length of a bear, than to weigh it. Data on 103 bears were recorded. For each bear, its weight, sex, length of body, girth of chest, girth of neck and month of measurement were recorded.

A Minitab analysis was performed to find the relationship between weight (in pounds) and two possible predictors, length of body (in inches) and girth of neck (in inches)

The following Minitab output was produced.

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Predictor        Coef       StDev          T        P
Constant      -307.57       24.24     -12.69    0.000
Length         2.5247      0.7350       3.44    0.001
Neck.G         16.317       1.371      11.90    0.000

S = 35.33       R-Sq = 90.6%     R-Sq(adj) = 90.4%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         2     1204782      602391    482.48    0.000
Residual Error   100      124853        1249
Total            102     1329635

Source       DF      Seq SS
Length        1     1027910
Neck.G        1      176872

Unusual Observations
Obs     Length     Weight         Fit   StDev Fit    Residual    St Resid
  2       47.5      70.00      138.70        9.24      -68.70      -2.01R
 19       63.0     140.00       22.82       16.33      117.18       3.74RX
 26       72.0     436.00      363.73        7.20       72.27       2.09R
 50       76.5     446.00      342.45        6.49      103.55       2.98R
 93       75.0     514.00      379.46        7.09      134.54       3.89R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.
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(a) Write down the fitted model. [1 mark]

Question 4 contd overleaf

Q4 contd

(b) Which is predicted to be heavier, a 5-foot tall bear with a neck girth of 27 inches or a 5ft 6” bear with a neck girth of 18 inches? [3 marks]

(c) Interpret fully the Minitab output. [14 marks]

(d) Suggest what further analysis you might do and why [2 marks]

5 (a) Let y_{ij} denote the jth observation in group i, $j = 1, \ldots, n_i$, $i = 1, \ldots, g$. Let $\bar{y}$ represent the overall mean and $\bar{y}_i$ represent the mean for group i. Given that $SS_{TOTAL} = \sum_i \sum_j (y_{ij} - \bar{y})^2$, $SS_B = \sum_i n_i (\bar{y}_i - \bar{y})^2$, $SS_W = \sum_i \sum_j (y_{ij} - \bar{y}_i)^2$ prove that $SS_{TOTAL} = SS_W + SS_B$.

[6 marks]

(b) A study of reading comprehension in children compared three methods of instruction, namely basal, DRTA and strategies. Children were randomly assigned to receive only one method of instruction. A score measuring comprehension skills was given to each child after a certain time. The scores recorded were as follows

Basal: 4, 6, 9, 12, 16, 15, 14, 12, 12, 8, 13, 9, 12, 12, 12, 10, 8, 12, 11, 8, 7, 9

DRTA: 7, 7, 12, 10, 16, 15, 9, 8, 13, 12, 7, 6, 8, 9, 9, 8, 9, 13, 10, 8, 8, 10

Strategies: 11, 7, 4, 7, 7, 6, 11, 14, 13, 9, 12, 13, 4, 13, 6, 12, 6, 11, 14, 8, 5, 8

Complete the ANOVA table below and test the null hypothesis that method of instruction has no effect on average score. State your conclusions. Is it appropriate to test for differences between pairs of groups? Justify your answer.

	SS	**d.f.**	**MS**	**F statistic**
Between groups				
Within groups				
Total	593.03			

d.f. = degrees of freedom, MS = mean square

[14 marks]

6. Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command may be modelled as a Normal random variable with standard deviation 25 milliseconds (ms). A new operating system has been installed. The network manager is planning a study to estimate the true average response time μ for the new environment.

(a) Why is the sample size important in such a study? [2 marks]

(b) Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% confidence interval for μ has a length of at most 10? [5 marks]

(c) The network manager wants to test the null hypothesis that the mean response time is 10ms. He considers a mean response time of 20ms to be the alternative of interest. Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the power of the hypothesis test is 90% at a significance level of $\alpha = 0.05$? [11 marks]

(d) For what alternative mean response times would the calculated sample size in (c) give a test of $H_0 : \mu_0 = 10$ of power at least 90% for $\alpha = 0.05$? [2 marks]

7. An engineer collects the following data on a sample of machine parts. The variable X recorded is the time to breakdown of the part, in days.

X: 32, 641, 410, 44, 206, 257, 547, 275, 738, 21, 135, 337, 306, 110, 97, 40, 41, 188

(a) Give reasons why, prior to observing any data, one might consider the variable 'time to breakdown' would ***not follow*** a Normal distribution. [3 marks]

(b) Letting X = time to breakdown, transform the data by letting $Y = \log_e(X)$. Draw a normal probability plot of these transformed data. Comment on whether the transformed data appear to be normally distributed. [11 marks]

(c) Estimate the mean time to breakdown from the plot. [4 marks]

(d) A 95% confidence interval for the mean value of $\log_e$ (time to breakdown) in the population is calculated to be (4.48, 5.58).

What are the difficulties in conveying these results to the engineer?

[2 marks]