1. (a) A company manufactures two types of computer, type A and type B. Manufacturing a type A computer requires 1 hour of labour and 2 computer chips. Each type B computer requires 2 hours of labour and 1 computer chip. There are 900 hours of labour and 500 chips available per month. The company is able to sell up to 200 type A computers and up to 180 type B computers per month. For each type A computer sold the company makes a profit of £200, while the profit on each type B computer is £150, and the company's aim is to maximise monthly profits. Formulate (but do not solve) this problem as a Linear Program. Ignore constraints that the variables are integers. [8 marks]

(b) In preparation for the winter season, a clothing company is manufacturing parka and goose overcoats, insulated pants, and gloves. All products are manufactured in four different departments: cutting, insulation, sewing, and packaging. The company has received firm orders for its products and will not produce over those orders. The contract stipulates a penalty for undelivered items. The following table provides the pertinent data of the situation.

	Time per unit (hr)				
Department	Parka	Goose	Pants	Gloves	Capacity (hr)
Cutting	.30	.30	.25	.15	1000
Insulating	.25	.35	.30	.10	1000
Sewing	.45	.50	.40	.22	1000
Packaging	.15	.15	.10	.05	1000
Demand	800	750	600	500	
Unit profit	£30	£40	£20	£10	
Unit penalty	£15	£20	£10	£8	

The target is to maximise the net receipts.

Formulate (but do not solve) this problem as a Linear Program. Ignore constraints that the variables are integers. [12 marks]

2. (a) Use the primal simplex method to solve the linear program

$$\text{maximise } x_0 = 3x_1 + x_2$$

subject to

$$\begin{aligned} 2x_1 + 3x_2 &\leq 60 \\ 2x_1 + x_2 &\leq 30 \\ x_1 &\leq x_2 + 3 \\ x_2 &\leq 18 \\ x_1, x_2 &\geq 0 \end{aligned}$$

[14 marks]

(b) Check all initial constraints for the optimal solution obtained. [2 marks]

(c) Formulate the dual linear program. [4 marks]

3. (a) Use the dual simplex method to solve the following linear program. (Solution by any other method will **not** receive credit.)

$$\text{maximise } x_0 = -x_1 - x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\geq 9 \\ x_1 - x_2 &\geq 6 \\ x_1 + 3x_2 &\geq 10 \\ x_1, x_2 &\geq 0 \end{aligned}$$

[12 marks]

(b) Sketch the feasible region for the same problem and find all the solutions graphically. [8 marks]

4. (a) Solve the following transportation problem starting from the initial basic feasible solution given.

	P	Q	R	S
A	[9]	[5] 11	[0]	[2]
B	[3] 8	[10]	[6]	[7] 7
C	[6] 5	[3] 4	[20] 2	[12]

[12 marks]

What is the cost of (i) the given initial solution; (ii) your optimal solution?

[2 marks]

(b) Explain how the unbalanced problem given below may be modelled as a balanced Transportation Problem (the storage costs at the sources equal zero).

	P	Q	R	Supply
A	[9]	[4]	[5]	20
B	[3]	[1]	[6]	20
C	[7]	[8]	[10]	12
Demand	15	25	10	

[3 marks]

Use the Least-Cost method to provide an initial basic feasible solution for the corresponding balanced Transportation Problem. [3 marks]

5. (a) Consider a maximisation bicriterion problem with objectives $Z_1(x)$ and $Z_2(x)$. What does it mean for a point y to be *inferior to* a point x? [2 marks]

Suppose different decisions result in the points in the objective space described as

$$0 \le Z_2 \le 9 - (Z_1 - 3)^2, \qquad 0 \le Z_1 \le 4.$$

Both objectives are to be maximised. Sketch the graph and indicate the non-inferior set (NIS). [4 marks]

(b) The vertices O, A, B, C, D of the feasible region for a bicriterion problem with two objectives Z_1 and Z_2 (linear functions of x, y which are to be maximised) are shown in the following table together with the corresponding values of the objectives.

Vertex:	O	A	B	C	D
(x, y):	(0, 0)	(1, 4)	(4, 2)	(4, 1)	(3, 0)
(Z_1, Z_2):	(0, 0)	(5, -7)	(6, 0)	(5, 2)	(3, 3)

Which of the points O, A, B, C and D corresponds to an inferior solution? In each such case give a solution to which it is inferior. What is the *Non Inferior Set* (or NIS) for this problem? [4 marks]

Let $Z(w) = (1 - w)Z_1 + wZ_2$. Determine, as a function of w for $0 \le w \le 1$, the set of points of the feasible region for which $Z(w)$ is maximised. [6 marks]

Suppose the goals for Z_1 and Z_2 are $G_1 = 13$ and $G_2 = 20$ respectively. Explain how to formulate a goal program in which the penalties for undershooting goals G_1 and G_2 are 4 and 3 per unit, respectively, and where there is no penalty for overshooting. [4 marks]

6. Consider the Main Convex Program:

minimise $f(x, y) = e^x + e^y$ subject to

$$g(x, y) = -x - y \le -b, \qquad x, y \ge 0,$$

where $b > 0$.

(a) Construct the Lagrangean $L(x, y, \lambda)$ for this problem. [2 marks]

(b) Write down all the necessary and sufficient conditions for a point (x, y, λ) to be a saddle point of the Lagrangean. [6 marks]

(c) Find the saddle point of the Lagrangean, in terms of b, and hence give the solution to the original Convex Program. [12 marks]

THE UNIVERSITY
of LIVERPOOL

7. For a single-item static continuous review model (Production-Inventory system) the Total Cost per Unit time, as a function of the number, y, of items made in each production run, is given by

$$TCU(y) = \frac{KD}{y} + \left(1 - \frac{D}{r}\right)\frac{hy}{2},$$

where K is the set up cost for a production run, r is the rate at which the item can be produced, D is the rate at which the item is used and h is the holding cost per item per unit time.

(a) Find the value of y for which $TCU(y)$ is minimized, and show for this value of y, namely $y = y^*$, that

$$TCU(y^*) = (2KDh(1 - D/r))^{\frac{1}{2}}.$$

[5 marks]

(b) Sketch a graph of stock level versus time. [3 marks]

(c) Calculate the values of y^* and $TCU(y^*)$ if $K = £40$ per order, $D = 25$ items per week, $h = £1$ per item per day, and $r = 40$ items per week. (We assume a week has 5 working days.) [3 marks]

Consider now the single-item static continuous review model above, but with $r = \infty$ (instantaneous replenishment).

(d) State the value of the Economic Order Quantity, y^*, in terms of K, D and h, and show in general

$$\frac{TCU(y)}{TCU(y^*)} = \frac{1}{2}\left(\frac{y}{y^*} + \frac{y^*}{y}\right).$$

[5 marks]

(e) Find the range of values of y (in terms of y^*) for which $TCU(y)$ is within 20% of its minimal value $TCU(y^*)$. [4 marks]